Singular Integral Operators with Integral Eigenvalues and Polynomial Eigenfunctions.

F. CALOGERO

Istituto di Fisica dell'Università - 00185 Roma

Istituto Nazionale di Fisica Nucleare - Sezione di Roma

(*Nuovo Cimento B*, **51**, 1 (1978))

A formulation of proposition 3 more precise than that given in the paper is the following one:

Proposition 3. If and only if

$$(2.14) \quad a(x) = (1-x^2)^{\frac{1}{2}} [1 + \alpha/(1-x) + \beta/(1+x)],$$

the operator $\mathcal{A}$ has, for every nonnegative integral value of n , a polynomial of degree n as eigenfunction. The corresponding eigenvalues are given by the simple formula

$$(2.15) \quad a_n = n, \quad n = 0, 1, 2, \dots$$

Analogous changes should be made wherever the results of proposition 3 are mentioned.

The previous formulation is unsatisfactory because it might appear to imply that there are no other eigenvalues and eigenfunctions. This, however, was not proven; indeed it is not true. ATKINSON (in a paper to be published) has in fact discovered that the operator $\mathcal{A}$ has also the (doubly degenerate) eigenvalue -1 , the corresponding eigenfunctions being $f_{\pm}(x) = (x-x_{\pm})^{-1}$, where $x_{\pm}$ are the 2 zeros of $a(x)$, $x_{\pm} = \frac{1}{2}\{a-\beta \pm \pm[(\alpha-\beta)^2 + 4(1+\alpha+\beta)]^{\frac{1}{2}}\}$. Note that for $\alpha > 0$, $\beta > 0$, there obtains $x_- < -1$, $x_+ > 1$, so that $f_{\pm}(x)$ are nonsingular in the interval $-1 \leq x \leq 1$.

Moreover, note that the adjective «ortogonal» in the second line of sect. 4 should clearly be omitted (see *Remark 2.3*).

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