Effective Potential, Instabilities and Bound-State Formation. Addendum.

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It was erroneous on our part to have identified the value of φ_c which minimizes V with the point at which $T(\varphi_c)$ has a zero-mass bound state. The origin of the error comes from taking the leading logarithmic approximation (1.1a) too seriously. In the vicinity of the bound state where $T \sim \varphi_c - \varphi_{c,B}$ it is illegitimate to approximate V by $\varphi_c^4 T$; rather V will tend to a limiting constant at the pole. The role of (1.1a) is to yield a perturbation-theoretic estimate of $\varphi_{c,B}$ and the residue, whereas the analytic form of V near the bound state is independent of the approximation.

The correct interpretation of $\varphi_{c,B}$ is in the variational inequality $(\varphi_c)_{\text{true}} > \varphi_{c,B}$. $\varphi_{c,B}$ is the limit of «longitudinal metastability». Longitudinal means along the direction of symmetry break-down and metastability limit means the limit of the domain of required positivity of response functions. Indeed, elementary stability considerations require positivity of $\lambda - T(\varphi_c)$ since the latter is a response function. The pole at $\varphi_c = \varphi_{c,B}$ is the limit of this domain.

The true value of φ_c can be found either from a direct determination of the minimum of $V(\varphi_c)$ or by locating the Goldstone pole (limit of transverse stability). Since the induced φ_2 mass is proportional to $\varphi_2 \varphi_2 \rightarrow \varphi_1 \varphi_1$ scattering and since the latter has the same t -dependence as in $\varphi_1 \varphi_1 \rightarrow \varphi_1 \varphi_1$ scattering (by gauge symmetry), we deduce that, at least in (1.1a), the true value of φ_c is at the zero of $T(\varphi_c)$. Interestingly enough this leads back to the first estimate of COLEMAN and WEINBERG using the Born approximation (i.e. T estimated to $O(\lambda)$ and $O(e^4)$).

It is important to point out the particularity of the present problem, a sort of dynamical break-down of symmetry but involving scalar mesons. The more usual mode of dynamical break-down is through nonvanishing scalar densities which are composite and a Goldstone meson which is a bound state. In these cases the true order parameter is obtained from the zero-momentum pole of scattering in the «Goldstone channel» and not the zero.

We take the occasion of this erratum to make an addendum concerning the gauge problem and the t -dependence of $T(\varphi_c)$, which we now understand more clearly than in our original write-up. It is easily checked that eq. (1) is the solution of the following

differential equations in t :

$$(a) \quad dT/dt = -3(d\pi/dt)T^2 - 3(d\pi_\gamma/dt)R^2,$$

$$(b) \quad dR/dt = -3(d\pi/dt)RT + 9(d\pi_\gamma/dt)RS,$$

$$(c) \quad dS/dt = (d\pi/dt)R^2 + 9(d\pi_\gamma/dt)S^2,$$

where S , R , T are light-by-light scattering, Compton scattering and 4-point ϕ scattering amplitudes respectively.

When gauge invariance is taken into account neither R nor S has t -dependence since this would require the corresponding gauge-noninvariant counter terms. When this is taken into account the integral of eq. (a) yields the tangent function eq. (3).