Communications

Elastic Stress at a Particle-Matrix Interface

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THE generation of interfacial stress between particles and matrix with dissimilar thermal expansion coefficients has been recently studied and reported by Olsen *et al.*¹ The system under investigation consisted of thoria particles dispersed in a nickel matrix. Since nickel has the higher expansion coefficient the possibility arises that a sufficiently large rise in temperature of the alloy can induce a tensile stress which exceeds the cohesive strength of the metal-ceramic interface. The development of this condition is favored by a rapid heating rate since this diminishes the effect of time-dependent stress relaxation processes. In their experiments Olsen *et al.* employed heating rates of $\geq 7000^\circ\text{C}$ per sec.

From their experimental technique Olsen *et al.* were able to determine a minimum temperature rise which could be associated with decohesion. Using this temperature change they calculated the interfacial strength of the alloy. It is the purpose of this note to show that the basis of this calculation is inaccurate, and leads to an underestimate of cohesive strength of approximately 25 pct.

Where α_p and α_m are respectively the linear thermal expansion coefficients of particle and matrix material the differential thermal expansion, δ , which develops on raising the temperature by an amount ΔT is given by:

$$\delta = (\alpha_p - \alpha_m) \Delta T$$

In the absence of relaxation processes producing plastic strains, this differential thermal strain is accommodated by elastic straining of both the particle and matrix. If σ (defined as positive if tensile) is the induced normal pressure at the interface then a spherical particle of radius r suffers a uniform hydrostatic dilation (dr/r) given by:

$$(dr/r) = \sigma/3K_p$$

where K_p is the bulk modulus of the particle. The ensuing tensile hoop strain ϵ in the matrix is then:

$$\epsilon = \sigma/3K_p + \delta \quad [1]$$

The Mott and Nabarro² formula for ϵ is:

$$\epsilon = \frac{3K_p \delta}{2K_p + (2E_m/1 + \nu_m)} \quad [2]$$

where E_m and ν_m are respectively Young's modulus and Poisson's ratio of the matrix. It follows from Eqs. [1] and [2] that:

$$\sigma = \frac{-6E_m K_p \delta}{2E_m + 3K_p(1 + \nu_m)} \quad [3]$$

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The negative sign means that the interface is under compression when the particle has the higher expansion coefficient (*i.e.* $\delta > 0$). This relationship for interfacial pressure is identical to the one quoted by Oriani³ for the analogous situation of coherent precipitates, where the strain δ arises from the distortion of atomic planes across the interface. Eq. [3] is however significantly different from the corresponding expression derived by Olsen *et al.*

Taking the experimentally determined figure of 306°C as the average temperature rise required to cause fracture, an interfacial cohesive strength of the nickel alloy of $\sim 40,400$ psi is deduced from Eq. [3]. The numerical data of Olsen *et al.* is used in the calculation; namely $\alpha_p = 9 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$, $\alpha_m = 13.5 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$, $E_m = 30 \times 10^6$ psi, $K_p = 28.7 \times 10^6$ psi and $\nu_m = 0.31$. On the basis of this data the above authors calculated a figure of 30,100 psi which appears to be ~ 25 pct too low.

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1. R. J. Olsen, G. Judd, and G. S. Ansell: *Met. Trans.*, 1971, vol. 2, p. 1353.

2. N. F. Mott and F. R. N. Nabarro: *Proc. Phys. Soc.*, 1960, vol. 52, p. 86.

3. R. A. Oriani: *Acta Met.*, 1964, vol. 12, p. 1399.

Discussion of "Elastic Stress at a Particle-Matrix Interface"

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In the paper by Olsen *et al.*⁴ the presence of lateral strains, $\epsilon_{\theta\theta}$ and $\epsilon_{\psi\psi}$, is neglected as a first approximation of the calculation for the stress across the interface. This approximation leads to an underestimation of 25 pct as correctly shown by Speight and Lobb.⁵ A more appropriate value for the interface strength can be calculated if the lateral strains are included as shown below.

The strains in the matrix at a distance r from the center of the particle, accompanying a differential thermal expansion, with the usual notation in spherical coordinates, are given by:⁶

$$e_{rr} = \frac{2\epsilon a^3}{r^3}, \quad e_{\theta\theta} = e_{\psi\psi} = -\frac{\epsilon a^3}{r^3}$$

where

$$\epsilon = \frac{3K_p \delta}{3K_p + 2E_m/1 + \nu_m}$$

and δ , the accommodated mismatch due to differential thermal expansion is

$$\delta = \left(\frac{1 + \alpha_p \Delta T}{1 + \alpha_m \Delta T} - 1 \right)$$

*M. V. SPEIGHT and R. C. LOBB: *Met. Trans.*, 1972, vol. 3, p. 737.

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