Addendum

On the Kuznetsov-Bruggeman formula for a Hilbert modular surface having one cusp

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There is a gap in the proof of

$$\sum_{\substack{c \\ 0 < c < T_1 \\ 0 < \varepsilon < T_2}} \frac{S(v, v', (c))}{N(c)} \ll_{v, v', \varepsilon} (T_1 T_2)^{1/6 + \varepsilon}$$

(Lemma 4.26) in the above-mentioned article. The proof of this estimate implicitly assumed that there are no exceptional eigenvalues for $SL_2(\mathcal{O}_K) \backslash H^2$. The exceptional eigenvalues for $SL_2(\mathcal{O}_K) \backslash H^2$, if any, create additional terms in this estimate. The correct result is

Lemma. Assume the notation of the above-cited paper. We have

$$\begin{aligned} \sum_{\substack{c \\ 0 < c < T_1 \\ 0 < \varepsilon < T_2}} \frac{S(v, v', (c))}{N(c)} &\ll_{v, v', \varepsilon} (T_1 T_2)^{1/6 + \varepsilon} \\ &+ \sum_{\substack{j \\ 0 < \lambda_j^{(1)} < \frac{1}{4} \\ 0 < \lambda_j^{(2)} < \frac{1}{4}}} [(T_1 T_2)^{2|r_{1j}| + \varepsilon} + (T_1 T_2)^{2|r_{2j}| + \varepsilon}]. \end{aligned}$$

Proof. The proof is exactly the same except that the estimate (4.30), which implicitly assumes that there are no exceptional eigenvalues, must be replaced by

$$\begin{aligned} \sum_{\substack{r_{1j} \leq M_1 \\ r_{2j} \leq M_2}} \left| \frac{\rho_j(v) \overline{\rho_j(v')}}{\cosh(\pi r_{1j}) \cosh(\pi r_{2j})} h(r_{1j}, r_{2j}) \right| \\ \ll (M_1 M_2)^{1/2} + \sum_{\substack{j \\ 0 < \lambda_j^{(1)} < \frac{1}{4} \\ 0 < \lambda_j^{(2)} < \frac{1}{4}}} [(T_1 T_2)^{2|r_{1j}| + \varepsilon} + (T_1 T_2)^{2|r_{2j}| + \varepsilon}]. \end{aligned}$$

This follows from the estimates of bump functions in the article of Deshoullier and Iwaniec, *Invent. Math.* **70**, pp. 270–273 (1982).