Acknowledgment of Priority Concerning: Polytopes that Fill $\mathbb{R}^n$ and Scissors Congruence

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Through the book of Engel [3, Theorem 9.11], we recently became aware of prior work on the problem treated in our paper [6].

The problem of proving that any polytope P that tiles $\mathbb{R}^n$ is scissors congruent to an n -cube was originally raised as an unsolved problem by Hadwiger [5] in 1963.

For Euclidean scissors congruence it was solved in dimensions 3 and 4 by Debrunner [2] in 1980. He proved that the codimension 2-Dehn invariant $\Delta(P) = 0$. He observed that $\Delta(P) = 0$ holds more generally for any polytope P such that $\mathbb{R}^n$ can be tiled using as prototiles similar polytopes $\alpha_i P$, where all the similarity ratios α_i are bounded above and below, i.e., there are constants c and C with $0 < c \leq \alpha_i \leq C < \infty$. Furthermore, the result is still valid if the tiling covers $\mathbb{R}^n$ except for finitely many “holes,” each of which is a bounded polytope.

For translation scissors congruence, Hadwiger [4] had already settled the case $n = 3$ in 1952. Mürner [7] settled the case for all n in 1975.

Thus Theorems 1 and 3 of [6] are not new, and the proofs in [6] are essentially the same as those of Debrunner [2] and Mürner [7], respectively.

As far as we know, Theorem 2 of [6] is new. We also reiterate the observation that the result that for any space-filling polytope the Dehn invariants $\Delta_{n,j}(P) = 0$ for $1 \leq j \leq n - 1$ applies to polytopes of arbitrary topological type, for example, to knotted space-filling polytopes, see [1].

References

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