Corrections to :

On Integrality of Eisenstein Liftings

Shin-ichiro Mizumoto

Mathematisches Institut, Universität Heidelberg, Im Neuenheimer Feld 288,
D-69120 Heidelberg, Germany*

and

Department of Mathematics, Tokyo Institute of Technology, Oh-okayama,
Meguro-ku, Tokyo, 152, Japan

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In my paper [Mi], the proof of Theorem 1.1 on pp. 209–211 was not sufficient: it applies only to $T(\ell)$'s defined below. So the paper should be corrected as follows.

For $\ell \in \mathbf{Z}_{>0}$, let

$$O_n(\ell) := \left\{ g \in \mathbf{Z}^{(2n)} \mid {}^t g J_n g = \ell J_n \right\}$$

and

$$T(\ell) := \sum_{\nu(g)=\ell} \Gamma_n g \Gamma_n \in L_{\mathbf{Z}}^{(n)},$$

where the summation is taken over the double cosets $\Gamma_n g \Gamma_n$ with $g \in O_n(\ell)$. Let p be a prime number and let

$$T_{j,n-j}(p^2) := \Gamma_n \left(\text{diag}(1_j, p 1_{n-j}, p^2 1_j, p 1_{n-j}) \right) \Gamma_n \in L_{\mathbf{Z}}^{(n)}$$

with $0 \leq j \leq n-1$, where diag denotes the matrix with indicated blocks in diagonal and 0 in the other blocks. The argument in [An, p. 149, Theorem 3.3.23] works over $\mathbf{Z}$, hence

$$L_{\mathbf{Z}}^{(n)} = \mathbf{Z} \left[T(p), T_{j,n-j}(p^2) \mid p : \text{prime}, 0 \leq j \leq n-1 \right].$$

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Let $\mathbf{T}'_{\mathbf{Z}}$ be the subalgebra of $\mathbf{T}_{\mathbf{Z}}$ generated over $\mathbf{Z}$ by

$$\{t_k^n(T(\ell)) \mid \ell \in \mathbf{Z}_{>0}\} \cup \{p^n \cdot t_k^n(T_{j,n-j}(p^2)) \mid p : \text{prime}, 0 \leq j \leq n-1\}.$$

Then $\mathbf{T}'_{\mathbf{Z}} \otimes_{\mathbf{Z}} \mathbf{Q} = \mathbf{T}_{\mathbf{Q}}$.

Claim. *Theorem 1.1 in [Mi, p. 209] is valid for $k > n$; but in case $k = n$, one should replace $\mathbf{T}_{\mathbf{Z}}$ by $\mathbf{T}'_{\mathbf{Z}}$.*

Proof. We need only to check the action of $t_k^n(T_{j,n-j}(p^2))$ on the Fourier coefficients, since the action of $t_k^n(T(p))$ has already been studied. As in [Zar, (2.2)], there exists a complete system of representatives of $\Gamma_n \backslash T_{j,n-j}(p^2)$ consisting of matrices of the form

$$\begin{pmatrix} p^2 D_{a,b}^{-1} & B \\ 0 & D_{a,b} \end{pmatrix} \begin{pmatrix} {}^t u^{-1} & 0 \\ 0 & u \end{pmatrix},$$

where

$$D_{a,b} := \text{diag}(1_{n-a-b}, p1_a, p^2 1_b) \quad \text{with } a, b \in \mathbf{Z}_{\geq 0} \text{ satisfying } a + b \leq n,$$

$$u \in (\text{SL}_n(\mathbf{Z}) \cap D_{a,b}^{-1} \text{SL}_n(\mathbf{Z}) D_{a,b}) \backslash \text{SL}_n(\mathbf{Z}),$$

and B runs over a complete system of representatives of

$$\left\{ B \in \mathbf{Z}^{(n)} \mid \begin{pmatrix} p^2 D_{a,b}^{-1} & B \\ 0 & D_{a,b} \end{pmatrix} \in T_{j,n-j}(p^2) \right\} / \text{mod } D_{a,b} \quad (*).$$

An element $M \in O_n(p^2)$ belongs to $T_{j,n-j}(p^2)$ if and only if $r_p(M) = j$, where $r_p(M)$ is the rank of M modulo p over $\mathbf{Z}/p\mathbf{Z}$. In particular we have $a > 0$. Let $N \in \mathbf{A}_n$ and $G_{a,b}(N) := \sum_B \mathbf{e}_n(N B D_{a,b}^{-1})$, where B runs over the same set as in (*). A similar argument as in [Mi, p. 210] shows that the claim follows from:

$$G_{a,b}(N) \in \mathbf{Z}$$

and

$$\text{ord}_p(G_{a,b}(N)) + 2(nk - \frac{n(n+1)}{2}) - (a+2b)k + n\delta_{k,n} \geq 0 \quad \text{for } k \geq n,$$

where ord_p denotes the p -order. To prove these assertions, we argue as in [An, pp. 157–158] to obtain

$$G_{a,b}(N) = 0 \quad \text{or} \quad p^{b(a+b+1)} S$$

with

$$S = \sum_C \mathbf{e}_a(p^{-1} N_{22} C),$$

where $N_{22} := N[\text{diag}(0_{n-a-b}, 1_a, 0_b)]$ and C runs over all integral symmetric matrices modulo p of size a satisfying $r_p(C) = j + a - n$. By [An, p. 244,

Lemma 4.2.25], we see $S \in \mathbf{Q}$, hence $S \in \mathbf{Z}$ since S is an algebraic integer. Thus the assertions follow. $\square$

Thus the following corrections should be made in [Mi]:

(1) In the proof of Theorem 1.1 on pp. 209–211, one should assume that $\nu(g)$ is a prime. With the above proof of the claim, Remark 1.3 on p. 211 remains valid.

(2) If $k = n$, $\mathbf{T}_{\mathbf{Z}}$ should be replaced by $\mathbf{T}'_{\mathbf{Z}}$ in the following 4 places:

p. 209: the statements of Theorem 1.1 and Corollary 1.2;

p. 221: the definition of $\kappa(f)$ in (4.4);

p. 222, line 6.

Hence in Theorem 6.5 on p. 229, one should employ the corrected definition of $\kappa(f)$ (which is a factor of $\mathcal{A}(f)$ as in (4.4)) provided $k = r$.

(3) There is a typographical error on page 223, line 10 from the bottom (Theorem 5.1(3)): $A_k^{(\nu)}\mathbf{Z}$ should be replaced by $A_k^{(m)}\mathbf{Z}$.

With the above corrections, the assertions in the paper, including the numerical examples in Sect. 7, remain true.

References

- [An] Andrianov, A. N., *Quadratic Forms and Hecke Operators*, Grundlehren der math. Wiss., vol. 286, Springer, 1987
- [Mi] Mizumoto, S., *On integrality of Eisenstein liftings*, Manuscripta Math. **89** (1996), 203–235
- [Žar] Žarkovskaya, N.A., *On the connections of the eigenvalues of Hecke operators and the Fourier coefficients of eigenfunctions for Siegel's modular forms of genus n* , Math. USSR Sbornik **25** (1975), 549–557; English transl