“EMBEDDINGS OF 2-SPHERES IN 4-MANIFOLDS” ERRATUM

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Theorem 2.4 of [A] is incorrect as stated there in the case of an odd lattice. The error is rooted in the preparatory Proposition 2.3. We came to realize the error while working on a generalization of Theorem 2.4 of [A] which will appear in the MPI-preprint series. In the case of an odd lattice, to characterize the orbits of the pairs studied, in addition to T_{α_1, α_2} , one needs the type of $\Lambda = \prec \alpha_1 \succ^\perp \cap \prec \alpha_2 \succ^\perp$ (i.e. even or odd) and in case Λ is even, the parity of $\alpha'_1 \cdot \alpha'_1$ for any α'_1 such that $\alpha'_1 \cdot \alpha_2$ is a canonical representative mod d . So the correct statement for an odd lattice should read:

Theorem 0.1. *Let $(L, \cdot)$ be an odd lattice whose rank r and signature σ satisfy: $r - |\sigma| \geq 4$. A complete set of invariants which characterize the orbit of a pair of mutually orthogonal ordinary elements is the "quintuple" T_{α_1, α_2} together with the type of Λ and the parity of $\alpha'_1 \cdot \alpha'_1$ for any α'_1 with $\alpha'_1 \cdot \alpha_2$ a canonical representative mod d , when Λ is even.*

Proof As in [A] we treat the problem in X_3 first and divide it into two cases: I. At least one element (say α_1) is of odd square. II. Both are of even square. Using [W] both cases contain more than one (as we mistakenly wrote in [A]) starting pairs :

$$\text{I: } \alpha_1 = (a+1)u_1 + av_1, \quad \alpha_2 = k(au_1 + (a+1)v_1) + lV,$$

with V one of a: $\gamma u_2 + (b+1)u_3 + bv_3$, $\gamma \in \{0, 1\}$ or b: $u_2 + v_2 + (2b+1)u_3 + (2b-1)v_3$.

$$\text{II: } \alpha_1 = -x_1 + ny_1, \quad \alpha_2 = k(x_1 + ny_1) + lV,$$

with V of the form: a: $x_2 + my_2$ or b: $(b+1)u + bv$ or c: $2x_2 + 2y_2 + (2b+1)u + (2b-1)v$. At the bottom of p.38 of [A], where case Ia is treated, there is an error. The coefficient d of part of α_2 is correct only when l is odd. Otherwise it is $2d$. We will use a different isometry to fix this. The rest of the treatment of Ia (p.39) is correct and we use it here. We treat all cases and show that different orbits within the same T_{α_1, α_2} are distinguished by the additional invariants introduced here. We begin with case I. Let $\phi_i = V_{13} \circ (12 - A^{-ia}) \circ V_{13} \circ U_{13} \circ (12 - A^{i(a+1)}) \circ U_{13}$ where U_{13} swaps u_1 and u_3 and V_{13} swaps v_1 and v_3 . Now by applying ϕ_1 to Ia followed suitably by 23- [W] (so as to get the pair at the bottom of p.38 of [A]) and finally the isometries on p.39 of [A] we get, for $\gamma \in \{0, 1\}$ a pair of the form:

$$(a+1)u_1 + av_1, \quad r(au_1 + (a+1)v_1) + d\{\gamma u_2 + (b+1)u_3 + bv_3\}, \quad (1)$$

For Ib begin by applying ϕ_1 followed by 23-[W] to get, for $W = u_2 + v_2 + (2c+1)u_3 + (2c-1)v_3$ or $u_2 + (c+1)u_3 + cv_3$ according as k, l are both odd or not:

$$(a+1)u_1 + a(v_1), \quad (k+2l)(au_1 + (a+1)v_1) + dW.$$

Take the second pair to one of the form (1) with $\gamma = 1$, working as in Ia. For the first, set $k+2l = dq + r$ and $(T, R) = ((q-i)/2, di+r)$, with $i = 0$ if q is even and

1 otherwise; apply $\phi_T \circ S_{u_2} \circ S_{v_2}$ followed by 23-[W] to get (for R, d both odd) :

$$(a+1)u_1 + a(v_1), \quad R(au_1 + (a+1)v_1) + d\{(2e+1)u_2 + (2e-1)v_2 + u_3 + v_3\}.$$

We now turn to Case II. Subcase IIa is done in [A]. Case IIb is done similarly to IIa, as follows: Apply first $a^n \circ e \circ b^{-1}$ followed by [W] in the direct sum of the second R_1 with X_1 to get a pair like IIa if $1/d$ is even and otherwise:

$$-x_1 + ny_1, \quad k(x_1 + ny_1) + d(x_2 + (c+1)u + cv).$$

Now finish the same way with case IIa contained in [A] (p.40) by "applying isometries 7-12 as in the even case" in R_2 followed by [W] in $R_1 \perp X_1$ to get:

$$-x_1 + ny_1, \quad r(x_1 + ny_1) + d((e+1)u + ev). \quad (2)$$

Now we deal with case IIc. When we apply $a^n \circ e \circ b^{-1}$ to IIc we get:

$$-x_1 + ny_1, \quad (k-2l)(x_1 + ny_1) + dZ,$$

with $Z = 2x_2 + 2y_2 + (2c+1)u + (2c-1)v$ or $x_2 + my_2$ according as $2n/d$ is even or not. The latter now is like case IIa. We treat the former. We will again essentially apply isometries 7 through 12 as in the even lattice case moving only within R_2 . Let $K = k - 2l = dq + r$. Apply $a^{T^n} \circ e \circ b^{-T}$ followed by [W] in $R_1 \perp X_1$ with $(T, R) = ((q-i)/2, di+r)$, $i = 0, 1$ according as q is even or odd. We get:

$$-x_1 + ny_1, \quad R(x_1 + ny_1) + d(2x_2 + 2y_2 + (2e+1)u + (2e-1)v).$$

So to summarize we have the following representatives:

$$\text{I: } (a+1)u_1 + a(v_1), \quad R(au_1 + (a+1)v_1) + d\beta,$$

where 1. $R = r$, $\beta = \delta u_2 + (c+1)u_3 + cv_3$ and $\delta \in \{0, 1\}$ or,

2. $\beta = (2e+1)u_2 + (2e-1)v_2 + u_3 + v_3$ and $R = r$ or $d+r$ and $R=\text{odd}$.

$$\text{II: } -x_1 + ny_1, \quad R(x_1 + ny_1) + d\beta$$

where 1. $R = r$, $\beta = x_2 + My_2 + \delta u$, $\delta \in \{0, 1\}$ or,

2. $\beta = 2x_2 + 2y_2 + (2e+1)u + (2e-1)v$ and $R = r$ or $d+r$ and $2n/d$ is even.

Now Cases I and II are distinguished by the parity of α_1^2 . Within Case I (resp. II), I1 and I2 (resp. II1, II2) are distinguished by the type of Λ : odd for I1 (resp. II1) and even for I2 (resp. II2). Within I1 (resp. II1) the value of δ and c (resp. δ and M) are determined by T_{α_1, α_2} and within I2 (resp. II2) the cases $R = r$ and $R = d+r$ are distinguished by the parity of $\alpha_1' \cdot \alpha_1'$ as in the statement of the theorem: even when $R = r$ and odd when $R = d+r$. This finishes the proof for X_3 . The problem in the case of a general odd lattice reduces to X_3 as in [A]. $\square$

REFERENCES

- [A] N. Askitas, *Embeddings of 2-Spheres in 4-Manifolds* Manus. Math. 89, 35-47, 1996
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