Corrigendum to the paper "HERMITIAN FORMS OVER DIVISION ALGEBRAS OVER REAL FUNCTION FIELDS"

Nguyen Quoc Thang

We make some corrections to the paper "Hermitian forms over division
algebras...", Bd. 78 (1993), pp. 9 - 35.

In the previous paper [T] we gave a classification of hermitian forms over the real function field $k = \mathbb{R}(t)$ and its completions k_v with respect to valuations v trivial on $\mathbb{R}$. Unfortunately in the local case the arguments given for cases A and D, in general, were not correct. Therefore the resulting local and local-global classifications obtained were incorrect. I would like also to thank Dr. D. Hoffmann for pointing out these mistakes and the referee for useful comments. Here we would like to make necessary corrections to [T]. We keep the same notation used there, except that in the first paragraph, J is not the standard involution of a quaternion division algebra D (with basis $\{1, i, j, ij\}$). All hermitian forms will be hermitian forms with respect to J , with values in D .

I. Local classification of forms of type D. Assume that D is of center k_v , $i^J = i, j^J = -j$. It is well-known (see [S2]) that, by scaling, forms of type D over D can be considered as hermitian forms with respect to J . By O_D (resp. p_D) we denote the valuation ring of D (resp. the maximal ideal of O_D); $O_D = \{d \in D : v(Nrd(d)) \geq 0\}$, $p_D = \{d \in D : v(Nrd(d)) > 0\}$. Let $\bar{D} = O_D/p_D$ and π (resp. t) be an uniformizing element of k_v (resp. of D , $t \in p_D$). We may assume that v is real. Then parts a), b), c) of Theorem 4.1.1 of [T] are true only in the case $D = (-1, -1)$ and incorrect in other cases, while part d) remains true. The reason is that we have used a result of [P], which is true only in the case $D = (-1, -1)$. Therefore the resulting Corollary 4.1.2 and Theorem 4.1.5 are true only in this case. Using the results of [S1], it can be shown that Theorems 4.1.1 and 4.1.5 should be replaced by the following.

Theorem 1. 1) Let $D = (-1, -1)$. Then $W^1(D, J) \simeq \mathbf{Z}/2\mathbf{Z} \times \mathbf{Z}/2\mathbf{Z}$. Any anisotropic J -hermitian form is equivalent to either $(i), (\pi i)$ or $(i, \pi i)$. A two-dimensional J -hermitian form F is isotropic if and only if $F \simeq (z, \lambda z)$, where $\lambda \equiv \pm 1 \pmod{k_v^{*2}}$ and z is a J -symmetric element of D . Any J -hermitian form of rank 3 represent any skew-quaternion of D .

2) Let $D = (\pi, -1)$. Then $W^1(D, J) \simeq \mathbf{Z} \times \mathbf{Z}/2\mathbf{Z}$. Any anisotropic J -hermitian form over D is equivalent to a form $(\epsilon, \dots, \epsilon) \perp pt(1)$, where $\epsilon = \pm 1$ and $p = 0$ or 1 .

Proposition 4.1.4 of [T] remains correct and from above it follows that Corollary 4.1.2 of [T] should be replaced by the following

Corollary. We have the following exact sequence of pointed sets

$$1 \rightarrow \mu_2 \rightarrow H^1(k_v, \mathrm{SU}(F)) \rightarrow H^1(k_v, \mathrm{U}(F)) \rightarrow E \rightarrow 1,$$

where $E = 1$ (resp. μ_2) if $D \simeq (-1, -1)$ (resp. $D \not\simeq (-1, -1)$).

II. Local classification of forms of type A. Let D be a quaternion division algebra over $K = k_v(\sqrt{a})$ with a K/k_v -involution J , where v is a real valuation. By Albert's Theorem [S2, Theorem 8.11.2, p. 314] we may assume that $D = (\alpha, -1) \otimes K$, where $\alpha \in k_v$. Thus D is a division algebra iff either $\alpha \equiv -1 \pmod{k_v^{*2}}$ or $\alpha \equiv -\pi \pmod{k_v^{*2}}$ and $a \equiv \pi \pmod{k_v^{*2}}$, where π is a fixed uniformizing element of k_v . Hence in all cases D/K is unramified, K/k_v is ramified and we may choose $t = \sqrt{\pi}$ to be an uniformizing element of K and D (see [T, p. 28]). Let O_v (resp. p_v) be the ring of v -integers of K (resp. maximal ideal of O_v). We use also the notation $\bar{J}$ and $\bar{J}_t$ as in [S1]. Theorem 5.1.1 of [T] is incorrect because $W^1(\bar{D}, \bar{J}) \simeq \mathbf{Z}$ (but not $\mathbf{Z}/2\mathbf{Z}$), while $W^1(\bar{D}, \bar{J}_t) \simeq \mathbf{Z}/2\mathbf{Z}$. Thus those statements concerning forms in Theorem 5.1.3 are incorrect also. Using the results of [S1], one can show that Theorems 5.1.1 and 5.1.3 should be replaced by the following.

Theorem 2. Any anisotropic J -hermitian form over D is equivalent to a form $(\epsilon, \dots, \epsilon) \perp \delta ti$, where $\epsilon \in \{\pm 1\}$, $\delta \in \{0, 1\}$ and $W^1(D, J) \simeq \mathbf{Z} \times \mathbf{Z}/2\mathbf{Z}$. If $\alpha \equiv -1 \pmod{K^{*2}}$ (resp. $\not\equiv -1 \pmod{K^{*2}}$) then $\mathrm{Nrd}(D^*)$

$= K^{*2}$ and $N_{K/k_*}(Nrd(D^*)) = k_*^{*2}$ (resp. $[k_* : N_{K/k_*}(Nrd(D^*))] = 2$). Therefore there are only four (resp. two) possibilities for determinants of forms of type A over k_* .

III. Local-global classification.

Theorems 4.2.1, 4.2.2, and 4.2.3 and Proposition 4.2.6 in the Local-Global Classification of [T] as well as their proofs remain correct, but the corrections in I and II necessitate the following alterations.

1) The correct formulation of Theorem 4.2.4 is as follows.

Theorem 3. *Let F be a form of type D of rank n with values in a quaternion division algebra over $k = \mathbf{R}(t)$.*

i) *If $D = (-1, -1)$ and $n \geq 2$ then F is isotropic.*

ii) *If $D \neq (-1, -1)$ and $n \geq 3$, then the Strong Hasse Principle holds for F .*

2) Regarding Proposition 4.2.7, one should add the condition " $D \simeq (-1, -1)$ " at the beginning.

3) Regarding Theorem 5.2.1, the last two sentences of b) should be removed due to corrections in II.

4) Regarding Proposition 5.2.2, the last sentence should be removed.

The proofs remain essentially the same.

References

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Nguyen Quoc Thang
 Department of Pure Mathematics
 University of Waterloo, Waterloo
 Ontario, Canada N2L 3G1
 E-mail: nqtnguyen@jeeves.uwaterloo.ca

(Received July 8, 1993;
 in revised form December 1, 1993)