Correction to An Isomorphism Theorem for Henselian Algebraic Extensions of Valued Fields

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In the example on page 123 of the article [1], the correct equation for π should be $\pi^{p-1} = -p$. As a consequence of the forgotten minus sign, the succeeding choice of the polynomial f and the corresponding verification is false. The intention was to provide a non-cyclic extension of $\mathbb{Q}_p(\varepsilon)$ of degree p , where $\varepsilon \neq 1$ denotes a p -th root of unity. The polynomial given in our example may be replaced by the polynomial $X^p + \pi X + \pi$. Let a be a root of this polynomial and let $\mathbf{K} = (\mathbb{Q}_p(\varepsilon, x), v)$ as in the example. Then $va = \frac{1}{p}v\pi \notin vK$ and consequently, the extension $\mathbf{K}(a)|\mathbf{K}$ is purely ramified; the residue field $\overline{K(a)}$ is equal to $\mathbb{F}_p$. Moreover, a is the only root of f in $K(a)$ (hence $K(a)|K$ is not cyclic). Indeed, assume that $b \neq a$ is another root of f . Then the element $z = \frac{b-a}{a}$ satisfies the equation

$$z^{p-1} + \sum_{i=0}^{p-2} \binom{p}{i+1} z^i + \frac{\pi}{a^{p-1}} = 0. \quad (1)$$

All binomial coefficients $\binom{p}{i+1}$ are divisible by p for $0 \leq i \leq p-2$. Moreover, $v\pi a^{-(p-1)} = pva - (p-1)va = va < vp$. Consequently, $vz = \frac{1}{p-1}va$ which is not an element of $vK(a)$ since $p > 2$. This shows that $b \notin K(a)$.

We are very indebted to F. Lorenz for giving us the idea how to find the right polynomial. His argument was as follows. Given a polynomial f , its discriminant is by definition a square in its splitting field. So we have to find an irreducible polynomial of degree p over $\mathbb{Q}_p(\varepsilon)$ whose discriminant is not a square in $\mathbb{Q}_p(\varepsilon)$. Indeed, since the extension by one of its roots a is of odd degree p , this discriminant will still not be a square in $\mathbb{Q}_p(\varepsilon, a)$, showing that $\mathbb{Q}_p(\varepsilon, a)$ is not the splitting field of f . This yields that the extension $\mathbb{Q}_p(\varepsilon, a)|\mathbb{Q}_p(\varepsilon)$ is not cyclic.

For polynomials of the form $X^n + cX + d$, the discriminant may be computed by the formula

$$\text{discr } f = (-1)^{n(n-1)/2} (n^n d^{n-1} + (1-n)^{n-1} c^n)$$

(cf. Lorenz [2], p. 316). With this formula, one finds that the discriminant of $X^p + \pi X + \pi$ is equal to $\pm(p^p \pi^{p-1} + (1-p)^{p-1} \pi^p)$. So $v \text{discr } f = p v \pi$ which is not divisible by 2 in $v \mathcal{O}_p(\varepsilon)$ and vK since $v\pi$ is the smallest element in both value groups.

Later in the same example, there is a paragraph which should better read as follows: "For every element $b \in L$ there exists some $\nu \in \{0, \dots, p^2 - 1\}$ and $c \in K$ such that $b^{-1} d_1^\nu c$ is a 1-unit, i.e. an element of $1 + \mathcal{M}_L$. Since $\pi_\delta^*(1 + \mathcal{M}_L) \subset \Theta_\delta^\times(\mathcal{O}_L^{2\delta})^\times$, this shows that $(\pi_\delta^* b)H = (\pi_\delta^* d_1^\nu)H$."

Finally, the compactness argument in Lemma 2.6 should be formulated in a slightly different way: "Since $L|K$ is algebraic, we have $L = \bigcup_{i \in I} K_i$ where $K_i|K$ runs through all finite subextensions of $L|K$. If every K_i admits a K -embedding σ_i in F then there is a K -embedding σ of L in F such that

$$\forall i \in I : \quad \sigma|_{K_i} = \sigma_j|_{K_i} \text{ for some } j \in I \text{ such that } K_i \subset K_j. \quad (2)$$

In particular, if all σ_i are valuation preserving, then so is σ ."

References

- [1] BASARAB, S. — KUHLMANN, F.-V.: An Isomorphism Theorem for Henselian Algebraic Extensions of Valued Fields. *manuscripta math.* **77**, 113 – 126 (1992)
- [2] LORENZ, F.: Einführung in die Algebra, Teil I. BI-Wiss.-Verl., Mannheim – Wien – Zürich (1987)

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