Addendum to „On Meromorphic Solutions of Algebraic Differential Equations”

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In this note, we make two additional remarks concerning the paper [1].

1. An easy modification of the proof in [1] can be used to establish the following important generalization of the main theorem in [1]:

THEOREM: *Let $\Lambda(z, y, dy/dz) = \sum Q_{kj}(z) y^k (dy/dz)^j$ be a polynomial in y and dy/dz whose coefficients are functions of z which are subject to the following restriction: If p is the total degree of Λ in y and dy/dz , let $Q_{kj}(z)$ be an entire function of finite order if $k+j < p$, while if $k+j = p$, let $Q_{kj}(z)$ be a polynomial. Then a meromorphic solution of the differential equation $\Lambda(z, y, dy/dz) = 0$ cannot be written as the quotient of two entire functions f/g , where f is of infinite order and g is of finite order. In particular, any entire solution, and more generally, any meromorphic solution whose sequence of poles has a finite exponent of convergence, must be of finite order of growth.*

2. Professor A. A. Gol'dberg has kindly informed the author that the question raised in [1], concerning the growth of meromorphic solutions of first order equations in the case when all coefficients $Q_{kj}(z)$ are polynomials, was solved in his paper [2], where it was shown that when all the coefficients are polynomials, then all meromorphic solutions must be of finite order. However, the author would like to point out that for the more extensive class of equations treated in Part 1 of this “Addendum”, the results obtained in [2] cannot be used to establish the theorem given in Part 1, since some coefficients are allowed to have essential singularities at ∞ , and, in fact, it is easy to see that such equations can possess meromorphic solutions of infinite order (e.g. $(\sin(e^z))^{-1}$ is a solution of the equation, $y^4 - y^2 - e^{-2z}(dy/dz)^2 = 0$).

REFERENCES

- [1] S. BANK, *On meromorphic solutions of algebraic differential equations*, Comm. Math. Helv., 44 (1969), 401–409.
- [2] A. A. GOL'DBERG, *On one-valued integrals of differential equations of the first order*, Ukrain. Mat. Ž., 8 (1956), 254–261 (Russian).

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