Correction to “On the characterization of flat metrics by the spectrum”

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In order to derive Proposition 5.3 on page 437, we must give a more precise definition of the weak C^∞ Riemannian structure on the normed space X .

Consider a map $x(\in X) \mapsto \beta(x)$ (the weak inner product of X) such that the topology of $X_{\beta(x)}$ does not depend on x . Set $\beta = \beta(0)$. If $x \mapsto \beta(x)$ is a C^∞ map of X into $L(X_\beta, X_\beta; \mathbf{R})$, we call this map a *weak C^∞ Riemannian structure* on X .

Under this definition, subsequent discussions are valid by changing Lemma 6.2 on page 438 as follows.

LEMMA 6.2. *Let $L_g = (1 + \bar{\Delta}_g)^2$, and $s > (n/2) + 3$, $s \geq k$. Then the maps*

$$\mathcal{R}^s \times S_2^k \rightarrow S_2^{k-4}; \quad (g, h) \mapsto L_g h$$

and

$$\mathcal{R}^s \times S_2^{k-4} \rightarrow S_2^k; \quad (g, h) \mapsto L_g^{-1} h$$

are of C^∞ class.

Proof. Since the differential operator $(1 + \bar{\Delta}_g)^2$ is an injective self-adjoint elliptic operator, L_g is an injective Fredholm operator from S_2^k to S_2^{k-4} for each $g \in \mathcal{R}^s$. First, we see that L_g has a continuous linear inverse $L_g^{-1}: S_2^{k-4} \rightarrow S_2^k$. For $k-4 \geq 0$, L_g is surjective by the decomposition theorem (e.g. [1, Ch. XI]). For a general k , $S_2^{k'}$ is a dense subspace of S_2^k if $k' \geq k$, hence $L_g(S_2^k)$ is dense in S_2^{k-4} from the above. On the other hand, $L_g(S_2^k)$ is closed because L_g is a Fredholm operator. Thus, L_g is surjective and accordingly has a continuous inverse by the open mapping theorem. Next, it is easily shown that the maps are of C^∞ class on the same lines as the proofs by Omori [2, Lemmas 1.3 and 2.11]. $\square$

REFERENCES

- [1] R. PALAIS, Seminar on the Atiyah-Singer index theorem, Ann. of Math. Studies, No. 57, Princeton University Press, Princeton, 1965.
- [2] H. OMORI, *On the group of diffeomorphisms on a compact manifold*, Proc. Sympos. Pure Math. 15, Amer. Math. Soc., 1970, 167–183.

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