Addendum to the Paper Entitled: **“Structural Theorems for Topological Actions of** **Z_2 -Torion Real, Complex and Quaternionic Projective Spaces”**

WU YI HSIANG

In the statement of Theorem 3, there is a missing case that all connected components of $F(G, X)$ are of RP -type. The condition for case 2 should be: $Sq^2(\zeta) = \gamma \cdot \zeta \neq 0$ and $Sq^1(\gamma) = 0$; and there should be the following third case, namely,

(c) Case 3, $Sq^2(\zeta) = \gamma \cdot \zeta \neq 0$ and $Sq^1(\gamma) \neq 0$; then all connected components of $F(G, X)$ are of RP -type, i.e., $F^j \sim RP^{(k_j-1)}$ and there exist $w_j, v_1, v_2 \in H^1(B_G)$ such that

$$\gamma = v_1^2 + v_1 v_2 + v_2^2, \quad \alpha_j = w_j (w_j + v_1) (w_j + v_1 + v_2)$$

and

$$i_j^*(\zeta) = \zeta_j^4 + \gamma \zeta_j^2 + (Sq^1 \gamma) \zeta_j + \alpha_j, \quad i_j^*: H_G^*(X) \rightarrow H_G^*(F^j).$$

And furthermore, the system of local weights at F^j, Ω_j , is given by

$$\Omega_j = \{(w_i + w_j), (v_1 + w_i + w_j), (v_2 + w_i + w_j), (v_1 + v_2 + w_i + w_j) \text{ multi. } k_i \ (i \neq j); \\ v_1, v_2, (v_1 + v_2), 0 \text{ multi. } (k_j - 1)\},$$

and

$$F(x) \text{ is of } \begin{cases} RP\text{-type} & \text{if } v_1 \mid G_x \text{ and } v_2 \mid G_x \text{ are linearly independent,} \\ CP\text{-type} & \text{if } v_1 \mid G_x \text{ and } v_2 \mid G_x \text{ are linearly dependent but not all} \\ & \text{zero,} \\ QP\text{-type} & \text{if } v_1 \mid G_x = v_2 \mid G_x = 0. \end{cases}$$

Correspondingly, one should add the following proof to the proof of Theorem 3 for the above case 3:

In the case $Sq^2(\zeta) = \gamma \cdot \zeta \neq 0$ and $Sq^1(\gamma) = \delta \neq 0$, it is not difficult to show that all connected components of $F(G, X)$ are of RP -type. Let ζ_j be the generator of $H^*(F^j)$, i_j^* be the restriction homomorphism of $H_G^*(X)$ to $H_G^*(F^j)$. Then, it follows from the following equations:

$$Sq^1(i_j^* \zeta) = i_j^*(Sq^1 \zeta) = 0 \quad \text{and} \quad Sq^2(i_j^* \zeta) = i_j^*(Sq^2 \zeta) = \gamma \cdot (i_j^* \zeta)$$

that

$$\begin{aligned} i_j^* \zeta &= \zeta_j^4 + \gamma \cdot \zeta_j^2 + \delta \zeta_j + \alpha_j; \\ Sq^1 \delta &= Sq^1 \alpha_j = 0, \quad Sq^2 \delta = \gamma \delta, \quad Sq^2 \alpha_j = \gamma \alpha_j. \end{aligned}$$

Let $f_j = (0, \dots, 0, \xi_j^{(k_j-1)}, 0, \dots, 0)$ be the fundamental cohomology class of F_j . Then, simple computation will show that the ideal $I_X(f_j)$ is generated by $a(f_j) = \delta^{(k_j-1)} \cdot \prod_{i \neq j} (\alpha_i + \alpha_j)$, (cf. the proof of Theorem 1). For simplicity, we may assume without loss of generality that $\alpha_1 = 0$.

(i) Suppose that at least one $k_j > 1$: Then it follows from Theorem B that δ splits into product of linear factors, say $\delta = v_1 \cdot v_2 \cdot v_3$. Notice that

$$Sq^1 \delta = (v_1 + v_2 + v_3) \cdot \delta = 0 \quad \text{implies} \quad v_1 + v_2 + v_3 = 0,$$

i.e.,

$$\delta = v_1 v_2 (v_1 + v_2),$$

and

$$Sq^2 \delta = \gamma \cdot \delta \quad \text{implies} \quad \gamma = v_1^2 + v_1 v_2 + v_2^2.$$

Again, it follows from Theorem B that $a(f_1) = \delta^{(k_1-1)} \cdot \prod_{j \neq 1} \alpha_j^{k_j}$ splits, and hence all $\alpha_j, j \neq 1$, split into product of linear factors, say

$$\alpha_j = w_j \cdot (w_j + \mu_{j,1}) (w_j + \mu_{j,2}) \cdot (w_j + \mu_{j,3}).$$

On the other hand,

$$Sq^1 \alpha_j = (\mu_{j,1} + \mu_{j,2} + \mu_{j,3}) \cdot \alpha_j = 0 \quad \text{implies that} \quad \mu_{j,1} + \mu_{j,2} + \mu_{j,3} = 0$$

$$Sq^2 \alpha_j = (\mu_{j,1} \mu_{j,2} + \mu_{j,2} \mu_{j,3} + \mu_{j,3} \mu_{j,1}) \cdot \alpha_j = \gamma \cdot \alpha_j$$

implies that

$$\{\mu_{j,1}, \mu_{j,2}, \mu_{j,3}\} = \{v_1, v_2, v_1 + v_2\}$$

as a set, namely

$$\alpha_j = w_j (w_j + v_1) (w_j + v_2) (w_j + v_1 + v_2).$$

(ii) Suppose all $k_j = 1$: Then $s = (n+1) > 1$, $a(f_1) = \prod_{j \neq 1} \alpha_j$. Hence, α_j again splits and the above proof will show that

$$\alpha_j = w_j (w_j + \mu_{j,1}) (w_j + \mu_{j,2}) (w_j + \mu_{j,3}), \quad \mu_{j,1} + \mu_{j,2} + \mu_{j,3} = 0,$$

and

$$(\mu_{j1}\mu_{j2} + \mu_{j2}\mu_{j3} + \mu_{j3}\mu_{j1}) = \gamma \quad \text{for all } j > 1,$$

and the assertion follows easily.

Remark. All actions of Case 3 are cohomologically modelled after the following type of linear actions on $QP^n = Sp(n+1)/Sp(1) \times Sp(n)$.

EXAMPLE 3c. Notice that the effective group of isometry on the symmetric space QP^n is $Sp(n+1)/\{\pm 1\}$ and $SO(3) = Sp(1)/\{\pm 1\}$ sits in $Sp(n+1)/\{\pm 1\}$ as diagonal unit quaternions. Let G be a $\mathbf{Z}_2$ -torus of $Sp(n+1)/\{\pm 1\}$ containing a maximal $\mathbf{Z}_2$ -torus of $SO(3)$. Then the restricted G -action on QP^n is of the type of Case 3.

Acknowledgement

The author wishes to thank Mr. Ted Chang and Mr. Tor Skjelbred for pointing out the possibility of the above third case.

Received January 16, 1975.