CORRECTIONS TO

“ CHARACTERISTIC FUNCTIONS SATISFYING A FUNCTIONAL EQUATION (II) ”

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The author have found some errors in his paper with the above mentioned title, published in the *Ann. Inst. Statist. Math.*, 21 (1969), 391-405.

1. In order to obtain (6.35), it was implicitly supposed that $\varepsilon_n(t) \leq \delta_n(A)/\sigma_n(A)$, which is not true (or at least needs proof). Lemma 9 and Theorem 7 should be omitted. The author is much indebted to Prof. B. Ramachandran for his attention to this point.

2. Equation (9.2), on which the proofs of Theorems 9-10 depend, is not true. So we replace them by

THEOREM 9'. *Suppose that a distribution F satisfies conditions (8.5) and (8.6) with $a_n = \gamma^{n/\alpha}$, $0 < \alpha < 2$. Then F belongs to the domain of partial attraction of a semi-stable distribution corresponding to (0.5). F has a moment $E(|X|^\beta)$ of positive order if $\beta < \alpha$, while $E(|X|^\beta) = \infty$ if $\beta > \alpha$.*

Outline of the proof. Derivation of (8.2)-(8.3) needs no change. (8.6) with $a_n = \gamma^{n/\alpha}$ implies that for sufficiently large n ,

$$(*) \quad F_0(a^{n+p+1}) = a^{-\alpha} F_0(a^{n+p})(1 + \varepsilon_p)$$

or

$$(**) \quad F_0(a^p x_0) = a^{-p\alpha} F_0(x_0)(1 + \varepsilon_p),$$

where $a = \gamma^{1/\alpha} = e^p (> 1)$, $x_0 \equiv a^n$, and $|\varepsilon_p| \leq \varepsilon$, $p = 1, 2, \dots$. Let ε be so small that $c \equiv (1 + \varepsilon)/a^{\alpha-\beta} < 1$ (when $\beta < \alpha$) and $d \equiv (1 - \varepsilon)/a^{\alpha-\beta} > 1$ (when $\beta > \alpha$). Then we have $(a^p x_0)^\beta F_0(a^p x_0) \geq x_0^\beta F_0(x_0) c^p$ (when $\beta < \alpha$) and, $(a^p x_0)^\beta F_0(a^p x_0) \leq x_0^\beta F_0(x_0) d^p$ (when $\beta > \alpha$). The last part of the Theorem follows from these inequalities. This together with (**), in turn, proves that under the assumption of the Theorem, (8.4) holds.

3. Corollary to Theorem 10 needs some more explanation.

$$\varphi(t) = \lim_{n \rightarrow \infty} \varphi^{[r^n]-1}(\gamma^{-n/\alpha} t) = \lim_{n \rightarrow \infty} \varphi^{[r^n]}(a^{-n} t) \quad \text{implies that} \quad \lim_{n \rightarrow \infty} [r^n](1 - F(a^{-n} x)) =$$

$-M(x)$ and $\lim [\gamma^n]F(-a^{-n}x) = N(-x)$, from which (8.5) and (8.6) with $a_n = \gamma^{n/\alpha} = a^n$ follow. Hence we have only to show that $E(|X|^\alpha) = \infty$. This follows from

$$a^{na}F_0(a^n) \sim [\gamma^n]F_0'(a^n) \rightarrow -M(1) + N(-1) > 0.$$

For further detailed discussion on this subject, see the subsequent paper "On the domain of partial attraction of semi-stable distributions" submitted to the present journal.