CORRECTION NOTE TO "ON MAXIMAL (k, t) -SETS"

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In the above paper (Ann. Statist. Math., 23(2), 279-292), Theorem 3.5, as stated, is incorrect. The correct statement of the theorem and its proof is given below:

THEOREM 3.5. $m_t(t, s) = t + 2$ for $s = t + 1$, where s is an odd prime or power of an odd prime.

PROOF. Suppose it is possible to choose $t + 3$ points in $PG(t - 1, s)$, no t linearly dependent. Carmichael has shown [2] that given a set of $t + 1$ points, no t linearly dependent, there exists a unique linear transformation which would take these $t + 1$ points into a set of points E_i ($i = 1, 2, \dots, t$) where E_i is a point having i th co-ordinate as unity and others zero and E has all co-ordinates as unity. In case of such a choice, there exists a matrix D whose columns give the co-ordinates of $t + 3$ points, no t linearly dependent.

$$(3.5.1) \quad D = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 1 & c_{11} & c_{12} \\ 0 & 1 & 0 & \dots & 0 & 1 & c_{21} & c_{22} \\ 0 & 0 & 1 & \dots & 0 & 1 & c_{31} & c_{32} \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \dots & 1 & 1 & c_{t,1} & c_{t,2} \end{bmatrix}.$$

Since the elements c_{ii} ($i = 1, 2, \dots, t$) can, without any loss of generality, be identified with the t non-zero elements of $GF(t + 1)$ in some order or the other, we may assume that

$$(3.5.2) \quad c_{ii} = w^{i-1} \quad i = 1, 2, 3, \dots, t$$

where w is a primitive element of $GF(s)$. This provides a set of at least $t + 2$ points in $PG(t - 1, s)$, no t linearly dependent. Hence, we have

$$(3.5.3) \quad m_t(t, s) \geq t + 2.$$

From the non-vanishing of the determinant formed by any $(t - 1)$ col-

umns from the first t columns and $(t+3)$ th column, it follows easily that $c_{i2} \neq 0$ for $i=1, 2, \dots, t$. Similarly, the elements c_{i2} , $i=1, 2, \dots, t$, cannot be unity.

Since c_{i2} , $i=1, 2, \dots, t$ need be distinct, these may also be identified with non-zero elements of $GF(t+1)$ in some order or the other. We may assume that $c_{12}=w^{i_0}$, $c_{22}=w^{i_1}$, $c_{32}=w^{i_2}$, $\dots$, $c_{t2}=w^{i_{t-1}}$ where $i_0, i_1, \dots, i_{t-1}$ is some permutation of $0, 1, 2, \dots, (t-1)$.

Since the determinant formed by $t-2$ columns from the first t columns and the last two columns is non-vanishing, it follows that

$$(3.5.4) \quad \frac{c_{12}}{c_{11}}, \frac{c_{22}}{c_{21}}, \dots, \frac{c_{t2}}{c_{t1}}$$

are all different and must be all non-zero multiples of $GF(s)$ in some order or other. Thus,

$$(3.5.5) \quad i_0, i_1, i_2, \dots, i_{t-1}$$

$$(3.5.6) \quad i_0, i_1-1, i_2-2, i_{t-1}-(t-1)$$

are two different permutations of $0, 1, 2, \dots, t-1$ when taken mod t . Let

$$(3.5.7) \quad g = 1+2+3+\dots+(t-1) = \frac{t(t-1)}{2}.$$

Now the sum of integers is, on one hand, congruent to $g \pmod{t}$ in (3.5.5) and on the other hand, congruent to $0 \pmod{t}$ in (3.5.6). It follows that

$$(3.5.8) \quad g \equiv 0 \pmod{t}.$$

This implies that (g/t) must be an integer. Since t is even, this is impossible. We have thus shown that

$$(3.5.9) \quad m_t(t, t+1) \leq t+2.$$

The assertion now follows from (3.5.3) and (3.5.9). QED

For $s=2^k$ and $s=t+1$, we give below the co-ordinates of $t+3$ points in $PG(t-1, s)$, no t linearly dependent.

$$(3.5.10) \quad \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & \dots & 0 & 1 & w & w^2 \\ 0 & 0 & 1 & \dots & 0 & 1 & w^2 & w^4 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 1 & w^{t-1} & (w^{t-1})^2 \end{bmatrix}.$$

Thus

$$(3.5.11) \quad m_i(t, s) \geq t+3 \quad \text{for } s=2^k, s=t+1.$$

This improves the lower bound due to Bush [1].

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- [1] Bush, K. A. (1952). Orthogonal arrays of index unity, *Ann. Math. Statist.*, **23**, 425-434.
- [2] Carmichael, R. D. (1937). *Introduction to the Theory of Groups of Finite Order*, Dover Publications, New York.