Erratum: Optimal Ray Sequences of Rational Functions Connected with the Zolotarev Problem

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Concerning the numerical methods presented in Section 7 of [1] for generating ray sequences of rational functions r_{mn} that are asymptotically optimal for the Zolotarev problem

$$Z_{mn} = Z_{mn}(E_1, E_2) := \inf_{r \in \mathbf{R}_{mn}} \left(\frac{\sup_{z \in E_1} |r(z)|}{\inf_{z \in E_2} |r(z)|} \right), \quad \frac{m}{n} \rightarrow \lambda,$$

the discussion of Leja–Bagby points in part (b) requires clarification. Indeed, as pointed out by V. Totik, the argument given on pages 259–261 of [1] does not, in general, cover the hypothetical case when the sequence of signed measures $\{\sigma_n\}$ has more than one limit measure. That is, we have actually established in [1] the following result.

Proposition 1. *Suppose the disjoint compacta E_1, E_2 are regular and $\lambda = p/q$ is rational. Set $\tau = \lambda/(\lambda + 1)$ and construct the Leja–Bagby points for the condenser (E_1, E_2) and corresponding rationals $r_n(z) \in \mathbf{R}_{np, nq}$ as in [1, p. 259]. If the associated sequence $\{\sigma_n\}_{n=1}^\infty$ of signed measures defined in (7.9) of [1] converges in the weak-star sense to the signed measure μ , then $\mu = \mu^*(\tau)$, where $\mu^*(\tau) = \mu_1^*(\tau) - \mu_2^*(\tau)$ is the extremal (equilibrium) measure given in [1, Theorem 3.1]. Consequently, the ray sequence $r_n(z)$ is asymptotically optimal for the Zolotarev problem (i.e., the limit relation following (7.10) of [1] holds).*

As we now show, the assumption that the full sequence of measures $\{\sigma_n\}_{n=1}^\infty$ converges can be removed if we form the Leja–Bagby points for the condenser (E_1^*, E_2^*) instead of (E_1, E_2) , where $E_i^* = E_i^*(\tau) := \text{supp}(\mu_i^*(\tau))$, $i = 1, 2$.

Proposition 2. *If the procedure for constructing Leja–Bagby described in Section 7(b) of [1] is applied on the support sets E_1^* and E_2^* of the extremal measure $\mu^* = \mu^*(\tau)$,*

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then $\sigma_n \rightarrow \mu^*$ in the weak-star topology. Consequently, the associated ray sequence of rationals $r_n(z) \in \mathbf{R}_{np,nq}$ is asymptotically optimal for the Zolotarev problem.

Proof. With the notation of [1], we proceed as on page 259 to obtain the estimate

$$\begin{aligned} (p+q)^{-2} \log \left(\frac{1}{A_{n+1}} \right) &= \tau \inf_{E_1^*} U^{\sigma_n} - (1-\tau) \sup_{E_2^*} U^{\sigma_n} \\ &\leq \int U^{\sigma_n} d\mu^* = \int U^{\mu^*} d\sigma_n = V_\tau, \end{aligned}$$

where the last equality follows since $\text{supp}(\sigma_n^{(1)}) \subset E_1^*$ and $\text{supp}(\sigma_n^{(2)}) \subset E_2^*$. The above inequality has the same form as the inequality (7.7) of [1] given for Fekete points, from which the convergence $\sigma_n \rightarrow \mu^*$ follows. ■

Remark 1. The above proof shows, more generally, that Leja–Bagby points for the sets

$$K_1(\tau) := \{z \in E_1 : U^{\mu^*}(z) = F_1(\tau)\}, \quad K_2(\tau) := \{z \in E_2 : U^{\mu^*}(z) = F_2(\tau)\},$$

which contain $E_1^*(\tau)$ and $E_2^*(\tau)$, respectively, have the described convergence properties. Moreover, it can be shown that for τ close to $1/2$ (λ close to 1), the Leja–Bagby points for the condenser $(K_1(\tau), K_2(\tau))$ are the same as the Leja–Bagby points for (E_1, E_2) and hence the latter have the desired convergence properties.

Remark 2. For the case of two intervals, we have found that Fejér–Walsh points appear to perform better than the corresponding Leja–Bagby points. However, the procedure for generating a convergent sequence of points can be implemented as follows. From Theorems 8.1 and 8.2, for $E_1 = [a_1, b_1]$ and $E_2 = [a_2, b_2]$, $b_1 < a_2$ and $1 > \tau > 1/2$, we know that $E_1^* = E_1$ and $E_2^* = [a_2, s]$, where $s = s(\tau)$. Since τ can be determined explicitly in terms of s , forming Leja–Bagby points for the pair of intervals $[a_1, b_1]$ and $[a_2, s]$, $a_2 < s < b_2$, yields a sequence that converges (by Proposition 2) to $\mu^* = \mu_\tau^*$.

The general problem of convergence of Leja–Bagby points for ray sequences on the condenser (E_1, E_2) remains an open problem.

Reference

1. A. L. Levin, E. B. Saff (1994): *Optimal ray sequences of rational functions connected with the Zolotarev problem*. Constr. Approx., **10**:235–273.

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