Errata Corrige

Boundary value problems for some degenerate-elliptic operators.

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The proof of the theorem (11.2) of part III in the above mentioned paper contains a faulty computation as was pointed out to us by NEIL S. TRUDINGER. The result however remains valid under an additional assumption on the coefficients. As most of the formal details are unchanged we shall indicate only necessary changes required.

The theorem in question concerns the HÖLDER continuity of variational solutions of linear degenerate-elliptic operators of the form

$$Au = -(a_{ij}u_{x_i} + d_j u)_{x_j} + b_i u_{x_i} + cu$$

for which there exists a measurable function $m = m(x) \geq 0$ with $m \in L^s(\Omega)$, $m^{-1} \in L^t(\Omega)$, $1/s + 1/t < 2/n$ (Ω being a bounded open set in $\mathbb{R}^n$) such that

$$a_{ij}(x)\xi_i\xi_j \geq m(x)|\xi|^2 \text{ a.e. in } \bar{\Omega} \text{ for all } \xi \in \mathbb{R}^n.$$

The other coefficients satisfy the hypothesis (A.1) and (A.2) of part II.

We recall that the proof of the theorem is based on the local estimates obtained on page 74 in theorem (8.5) where the constants C_8 , C_9 are functions of the norms of m and m^{-1} on balls of small radii and so are functions of the radius. More precisely, denoting by $B(x_0, r)$ the ball of centre x_0 and radius r , and by

$$N_{1, s}(m, r) = \left[(2r)^{-n} \int_{B(x_0, r)} m^s dx \right]^{1/s}$$

we have $C_8 = \text{Const. } N_{1, s}(m, 2r)^{\xi/\tilde{s}} N_{1, t}(m^{-1}, 2r)^{\xi/2} = \text{Const. } [|\text{vol. } B(x_0, 2r)|^{-(1/\tilde{s}s + 1/2t)} \|m\|_{L^s(B(x_0, 2r))}^{1/\tilde{s}} \|m^{-1}\|_{L^t(B(x_0, 2r))}^{1/2}]^{\xi}$ where the constant is independent of r and ξ is a

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numerical constant also independent of r . The constant C_9 corresponds to the case $s = \infty$ and hence $\tilde{2}$ is replaced by $2^\#$. On the other hand, for the proof of theorem (11.2) we require to choose an l_0 large enough so that for all $l \geq l_0$,

$$|r^{-n} \text{ meas. } E(x_0, 2r, k_l)|^{\theta/2} < (2C_8)^{-1}$$

where $\theta > 1$ is a numerical constant (see theorem (8.5)) and

$$k_l = M(x_0, 4r) - 2^{-(l+1)}(M(x_0, 4r) - m(x_0, 4r))$$

with $M(x_0, 4r)$ and $m(x_0, 4r)$ the maximum and minimum of the solution on the ball $B(x_0, 4r)$. Hence if C_8 were independent of r (so that the estimates of theorem (8.5) remain true under affine transformations) then it is possible to choose l_0 as above in view of proposition (11.1). Similar observations hold also for C_9 . These considerations lead us to make the following additional assumption:

(B) for any ball $B \subset \Omega$ the quantity

$$|\text{meas. } B|^{-(1/\tilde{2}s+1/2t)} \|m\|_{L^{\tilde{2}}(B)}^{1/\tilde{2}} \|m^{-1}\|_{L^t(B)}^{1/2}$$

is uniformly bounded.

The hypothesis (B) is essentially an assumption on the leading coefficients a_{ij} of A . In the non-degenerate uniform elliptic case which corresponds to $t = s = \infty$, this condition is always satisfied. However in the case where in $t = \infty$, $s \geq n/2$ the theorem (11.2) under the hypothesis (B) seems to be new.

In view of what has been said above the results of sections (11) and (12) remain true with the additional assumption (B).

A hypothesis of the above type occurs also in the forthcoming papers of D. E. EDMUNDS and L. A. PELETIER titled «A Harnack inequality for weak solutions of quasilinear elliptic equations» to appear in J. London Math. Soc. 1972, and of N. S. TRUDINGER titled «Linear elliptic operators with measurable coefficients» to appear in Ann. Sc. Norm. Sup. The authors gratefully acknowledge the observations made by professor TRUDINGER and for the preprint of the announcement of the above mentioned paper provided to us by him.