ERRATUM FOR
A COMPARISON OF SEVERAL k SAMPLE TESTS FOR ORDERED
ALTERNATIVES IN COMPLETELY RANDOMIZED DESIGNS

MARK L. BERENSON
BARUCH COLLEGE, CUNY

Due to a printer's error, the columns of Table 1 of Berenson [1982] were misaligned. The correct version of the Table is given on the pages that follow. (Ed.)

TABLE I
k-sample Test Procedures Used in Monte Carlo Study

Test Type of Test (Alternative)	Test Statistic	Decision Rule
F-Test: [Snedecor, 1962] Parametric (General)	$F = \frac{n \sum_{j=1}^k (\bar{X}_{.j} - \bar{X}_{..})^2 / (k - 1)}{\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{.j})^2 / (N - k)}$	Reject H_0 if $F \geq F_{\alpha(k-1, N-k)}$
Bartholomew's $\bar{E}^2$ Test: [Bartholomew, 1959a; 1959b; 1961a] Parametric (Ordered)	$\bar{E}^2 = \frac{n \sum_{j=1}^k (\hat{\mu}_j - \bar{X}_{..})^2}{\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{..})^2}$	<p>Reject H_0 if</p> $\sum_{l=2}^k p(l, k) I_{1-E^2} \left(\frac{N-l}{2}, \frac{l-1}{2} \right) \leq \alpha$ <p>where</p> $I_{1-E^2} \left(\frac{N-l}{2}, \frac{l-1}{2} \right)$

where $\hat{\mu}_j$ are the maximum likelihood estimates of μ_j obtained by an "amalgamation process" [Brunk, 1958] so that $\hat{\mu}_1 \leq \hat{\mu}_2 \leq \dots \leq \hat{\mu}_k$.

is the incomplete beta function [K. Pearson, 1956] with parameters $(N - l)/2$ and $(l - 1)/2$, and where $p(l, k)$ are the "combining probabilities" computed from the recurrence relation given by Miles [1959]:

$$p(l, k) = [p(l - 1, k - 1) + (k - 1)p(l, k - 1)]/k,$$

where

$$2 \leq l \leq k - 1; \quad p(1, k) = 1/k; \quad \text{and} \quad p(k, k) = 1/k!$$

Linear Regression
Test (t_L):
[Abelson and Tukey,
1963]

Parametric
(Ordered)

Reject H_0 if $t_L \geq t_{\alpha(N-k)}$

$$t_L = \frac{\sum_{j=1}^k c_j X_{\cdot j} / \left(n \sum_{j=1}^k c_j^2 \right)^{1/2}}{\left[\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{\cdot j})^2 / (N - k) \right]^{1/2}}$$

where c_j are the usual coefficients of orthogonal linear contrasts [Winer, 1971].

Abelson-Tukey Optimal
Contrasts (t_A):
[Abelson and Tukey,
1963]

Parametric
(Ordered)

Reject H_0 if $t_A \geq t_{\alpha(N-k)}$

$$t_A = \frac{\sum_{j=1}^k M_j X_{\cdot j} / \left(n \sum_{j=1}^k M_j^2 \right)^{1/2}}{\left[\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{\cdot j})^2 / (N - k) \right]^{1/2}}$$

where the "optimal contrast" coefficients M_j are given as

$$M_j = \{(j-1)[1 - ((j-1)/k)]\}^{1/2} - \{j(1 - (j/k))\}^{1/2}.$$

Jonckheere's S Test:
[Jonckheere, 1954]
Nonparametric
(Ordered)

Reject H_0 if

$$S \geq 1 + Z_{\alpha} [(N^2(2N+3) - k(n^2)(2n+3))/18]^{1/2}$$

provided that $N \geq \underline{\quad}$

$$S = 2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k P_{jm} - k(k-1)n^2/2,$$

where

$$P_{jm} = \sum_{i=1}^n \sum_{l=1}^m P_{ij,lm},$$

and where

$$P_{ij,lm} = \begin{cases} 1 & \text{if } X_{ij} < X_{lm} \\ 0 & \text{if } X_{ij} \geq X_{lm} \end{cases}$$

TABLE 1

k-sample Test Procedures Used in Monte Carlo Study, continued

Test Type of Test (Alternative)	Test Statistic	Decision Rule
Modified Cox-Stuart Weighted Sign Test (<i>W</i>): [Cox and Stuart, 1955; Berenson, 1974]	$W = \sum_{m=1}^{N/2} (N' - 2m + 1)\delta_{m, N-m+1}$	Reject H_0 if $W \geq \frac{N'^2}{8} + Z_\alpha \left[\frac{N'(N'^2 - 1)}{24} \right]^{1/2}$
Nonparametric (Ordered)	<p>where the indicator variable</p> $\delta_{m, N-m+1} = \begin{cases} +1 & \text{if } Y_m < Y_{N-m+1} \\ 0 & \text{if } Y_m \geq Y_{N-m+1} \end{cases}$ <p>and where the "weights" $N' - 2m + 1$ represent the distances between the two observations being compared; $N' = N$ if N is even, $N' = N - 1$ if N is odd, and the sequence X_{ij} is replaced by the sequence Y_i (where $i = 1, 2, \dots, N$) as follows:</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> $X_{11}, X_{21}, \dots, X_{n1}, X_{12}, X_{22}, \dots, X_{1k}, \dots, X_{nk}$ $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$ </div> <div style="text-align: center;"> $Y_1, Y_2, \dots, Y_n, Y_{n+1}, Y_{n+2}, \dots, Y_{(k-1)n+1}, \dots, Y_N$ $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$ </div> </div>	provided that $N > 15$; otherwise, use exact α levels for the null distribution of W .

<p>Chacko's $\bar{H}$ Test: [Chacko, 1963]</p> <p>Nonparametric (Ordered)</p>	$\bar{H} = \frac{12n}{N(N+1)} \sum_{j=1}^k \left[\bar{R}_j - \frac{N+1}{2} \right]^2$	<p>Reject H_0 if</p> $\sum_{i=2}^k p(l, k) P_r \{ \bar{H} \geq \bar{H}_c \} \leq \alpha$
<p>where $\bar{R}_j$ are the estimates obtained by an "amalgamation process" [Brunk, 1958] so that $\bar{R}_1 \leq \bar{R}_2 \leq \dots \leq \bar{R}_k$.</p>	<p>where the test statistic $\bar{H}$ is approximated by χ^2_{l-1} and where $p(l, k)$ are the "combining probabilities" given by Miles [1959]—see $\bar{E}^2$ test.</p>	
<p>Puri's Expected Normal Scores Test (V): [Puri, 1965; Berson, 1978]</p> <p>Nonparametric (Ordered)</p>	$V = \sum_{j=1}^k L_j \xi_{\cdot j}$ $L_j = \begin{cases} 2nc_j & \text{if } k \text{ is odd} \\ nc_j & \text{if } k \text{ is even} \end{cases}$ <p>where</p>	<p>Reject H_0 if</p> $V \geq Z_\alpha \left\{ \frac{n \sum_{i=1}^n \sum_{j=1}^k \xi_{ij}^2}{N-1} \left\{ \sum_{j=1}^k L_j^2 \right\} \right\}^{1/2}$
<p>and c_j are the usual coefficients of orthogonal linear contrasts [Winer, 1971].</p>	<p>provided that $N \geq 15$.</p>	
<p>Johnson-Mehrotra Expected Normal Scores Test (T^*): [Johnson and Mehrotra, 1971; Berson, 1976]</p> <p>Nonparametric (Ordered)</p>	$T^* = \sum_{j=1}^k M_j \xi_{\cdot j}$ <p>where the M_j are the Abelson-Tukey [1963] "optimal contrast" coefficients—see t_A procedure.</p>	<p>Reject H_0 if</p> $T^* \geq Z_\alpha \left\{ \frac{\sum_{i=1}^n \sum_{j=1}^k \xi_{ij}^2}{n(N-1)} \left\{ \sum_{j=1}^k M_j^2 \right\} \right\}^{1/2}$
	<p>provided that $N \geq 15$.</p>	