CORRECTION TO MY PAPER ON THE COLOURING OF INFINITE GRAPHS AND THE THEOREM OF KURATOWSKI

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(Presented by G. HAJÓS)

N. G. DE BRUIJN has pointed out in [1] that condition (iv) in § 2 of [4] is too weak and this invalidates a theorem which claims that (iv) is equivalent to some other conditions (i), (ii) and (iii) stated there. These conditions are stipulations on a graph G ; e.g. (ii) is the requirement that every finite subgraph¹ of G be homeomorphically imbeddable in the plane. The following corrected form of (iv) is equivalent to the other conditions.

(iv) There exists two functions I and E defined on the set of all circuits² of G whose values are subgraphs of G such that

$$(1) \quad I(C) \cap E(C) = C \quad \text{and} \quad I(C) \cup E(C) = G;$$

(2) if P_1 , P_2 and P_3 are three pathes³ of G with disjoint interiors but common ends then the three circuits $P_1 \cup P_2$, $P_1 \cup P_3$ and $P_2 \cup P_3$ can be ordered in a sequence C_0 , C_1 , C_2 such that

$$I(C_0) = I(C_1) \cup I(C_2) \quad \text{and} \quad I(C_1) \cap I(C_2) = C_1 \cap C_2.$$

The idea of this condition is that $I(C)$ and $E(C)$ in a sense replace the domain bounded by C and the exterior of this domain which would exist if G was imbedded in the plane. Now the proof of the equivalence of the conditions (i), ..., (iv) outlined in [4] becomes valid (after an obvious modification of the step (iii) $\rightarrow$ (iv)).

Several other combinatorial conditions for planar graphs are known (see e.g. [6], where other references are given), but (iv) differs from the others which in general do not seem to be applicable to the infinite graphs possibly of high power.

For additional facts and references concerning the subject of § 1 of [4] see [2], [3] and [5].

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¹ i.e. a collection of simplices of G which is a closed complex.

² i.e. subgraphs topologically equivalent to the circumference of a circle.

³ i.e. subgraphs topologically equivalent to a closed interval on the real line.

References

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