Guest Editor's Foreword

This issue contains four papers selected from those presented at the Fifth Annual Symposium on Computational Geometry held at Saarbrücken, West Germany, June 5–7, 1989. Preliminary versions of these papers appeared in the conference proceedings. The papers were selected for their originality and significance and for their relevance to the goals of this journal.

The four papers can be classified into two groups, although, on a higher level, they all share the common theme of *decomposition of geometric objects*. The first two papers consider the issue of “derandomization” in computational and combinatorial geometry. In the last few years there has been a considerable amount of work in randomized techniques in geometry. The introduction of epsilon nets into the field by Haussler and Welzl, and the alternative probabilistic techniques of Clarkson and of Clarkson and Shor, have facilitated a host of results in the area, where random sampling of the input data allows one to break a geometric problem into smaller well-behaved subproblems whose combined solutions yield the desired overall solution. Among problems that were successfully tackled by these techniques are simplex range searching, point location in higher dimensions, and combinatorial analysis of arrangements.

These randomized solutions have many additional advantages. They are relatively easy to implement, and are applicable in fairly general contexts. Moreover, alternative deterministic techniques could not match their performance. Still, the problem of finding deterministic solutions that would substitute for the randomized techniques has been one of the challenging problems in the area. Initial approaches to this problem have recently been given by Chazelle and Friedman, but the running time of their procedures was not very efficient.

The first two papers of this issue provide better, and much closer to optimal, solutions for the special case involving arrangements of lines in the plane. The main problem addressed in these papers is: given n lines in the plane, and a parameter $r < n$, find a decomposition of the plane into $O(r^2)$ triangles such that no triangle meets more than $O(n/r)$ of the lines. Previous randomized techniques for this problem involved random sampling of the given lines. The present papers present efficient deterministic solutions.

The first paper by Matoušek contains an initial innovative solution, which

blends many techniques for handling arrangements of lines, including computation of approximate levels in arrangements, techniques for approximately counting intersections between lines, and efficient deterministic construction of epsilon nets in arrangements of lines. The paper can clearly be regarded as a breakthrough in this area.

The first of a pair of companion papers by Agarwal (the second paper will appear in the next issue) is a follow-up and refinement of the ideas in Matoušek's paper. It provides an improved technique that, for large values of r , is about $O(r)$ times faster, and in some sense is almost optimal. The first part of the paper presents the refined decomposition algorithm and its analysis, and the second part presents many applications, all resulting in efficient deterministic algorithms for a variety of problems, such as computing many faces in arrangements, counting segment intersections, computing spanning trees with low stabbing number, and many others.

The third and fourth papers are different, and have a common theme of decomposing polyhedral scenes in three dimensions. They exemplify a recent trend in computational geometry of more intensive study of problems in three and higher dimensions. Most of the research in computational geometry has concentrated on problems in the plane, which of course were much easier to handle. Problems involving three-dimensional data received less attention, with the exception of several "classical" topics such as convex hulls or linear programming. The applications of problems in three dimensions are extensive, and arise in computer graphics, computer-aided design, robotics, computer vision, and more. The third and fourth papers of this issue make considerable contributions in this direction.

The third paper, by Paterson and Yao, considers the problem of binary space partitions, a method that has been earlier introduced as a heuristic technique for hidden surface removal and other rendering problems in graphics and solid modeling. Roughly speaking, the problem is: given n nonintersecting triangles in space, one wants to partition the space by a plane, that will cut as few triangles as possible, and then apply this step recursively within each resulting half-space. The goal is to minimize the overall number of pieces into which the original triangles have to be split. The paper provides several techniques to obtain such a partition with guaranteed worst-case behavior. Incidentally, it does make use of randomization.

The fourth paper, by Chazelle and Palios, considers the problem of triangulating a nonconvex polytope in three dimensions. This is also an important problem that arises in many applications, e.g., solid modeling, or motion planning within such a polytope. Since convex polytopes admit trivial linear-size triangulations, the goal is to obtain an efficient decomposition whose size depends nonlinearly only on the number of reflex edges of the polytope. The authors achieve such an efficient procedure, which produces a triangulation whose size is $O(n + r^2)$, where n is the number of faces of the polytope and r is the number of its reflex edges; this is known to be tight in the worst case up to a constant factor. The running time of their procedure is $O((n + r^2) \log r)$, the best among several recent attacks on the problem.

As is the practice in this fast growing field, considerable progress has been made since the original appearance of these papers. For example, further improvements and generalizations of the deterministic decomposition algorithms for arrangements have been recently obtained by Matoušek. And Paterson and Yao have improved their bounds on binary space partitions in the special case of rectilinear scenes. Alas, in this field, like Lewis Carroll's Red Queen, it takes all the running one can do to remain in the same place. . .

In closing these introductory notes, I wish to thank the authors for their contributions and the referees for their timely and reliable work. I am pleased to have the privilege of editing this special issue, and hope the reader will enjoy it.

MICHA SHARIR
Guest Editor