Erratum

Semi-Infinite Weil Complex and the Virasoro Algebra

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In Sect. 4, on p. 636, we had assumed an incorrect structure of the Fock representations $\mathcal{H}_p$ of the Virasoro algebra, taken from [1], Theorem 1.10 (cf. Fig. 5). In fact, the module $\mathcal{H}_p$ is isomorphic to the Verma module, if $p \leq 0$, and to the contragradient Verma module, if $p > 0$, with highest weight $h'_p = -(p-2)(p+1)/2$ and central charge 28 [2].

For this reason the exact sequences (24), (25), and Proposition 7 on p. 637 are also incorrect. Proposition 7 should read as follows.

- Proposition 7.** 1) Let $m \geq 0$, $n \leq 0$ be of equal parity. Then $h^j(L_m \otimes \mathcal{H}_n) = \delta_{j,0}$, if $m = -n$ and 0, otherwise.
2) Let $m \geq 0$, $n \leq 0$ be of different parity. Then $h^j(L_m \otimes \mathcal{H}_n) = \delta_{j,1}$, if $m = -n-1$ and 0, otherwise.
3) Let $m \geq 0$ and $0 < n < m$. Then $h^j(L_m \otimes \mathcal{H}_n) = 0$ for any j .

Proof of parts 1) and 2) follows from the isomorphism $\mathcal{H}_n \simeq M_{(h_n, 28)}$ for $n \leq 0$, Proposition 5, and the short exact sequence

$$0 \rightarrow L_x \rightarrow M_x^* \rightarrow M_{x+1}^* \rightarrow 0.$$

Part 3) can be proved in a similar fashion. $\square$

This corrects (and simplifies) the statement of Theorem 1 on p. 628.

- Theorem 1.** 1) Let $p = -2m$, $m \geq 0$. Then $h_p^{0,l} = 1$, if $l \leq m$, and $h_p^{j,l} = 0$, otherwise.
2) Let $p = -2m-1$, $m \geq 0$. Then $h_p^{1,l} = 1$, if $l \leq m$ and $h_p^{j,l} = 0$, otherwise.
3) Let $p = -2m+1$, $m \leq 0$. Then $h_p^{0,l} = 1$, if $l \geq m$, and $h_p^{j,l} = 0$, otherwise.
4) Let $p = -2m+2$, $m \leq 0$. Then $h_p^{-1,l} = 1$, if $l \geq m$ and $h_p^{j,l} = 0$, otherwise.

Proof. Let $p = -2m$, $m \geq 0$. By Theorem 4, $h_p^{j,l} = \sum_{k \geq 0} h^j(L_{|l|+2k} \otimes \mathcal{H}_{p+l})$. It is equal to $\delta_{j,0}$, if $l \leq m$, and 0, if $l > m$, by Proposition 7.

In other cases the proof is similar. $\square$

Since $d_p^{j,l} = h_p^{j,l} + h_p^{j-1,l}$ (cf. p. 628), Theorem 2 on p. 628 should read as follows.

Theorem 2. 1) Let $p = -2m$, $m \geq 0$. Then $d_p^{0,l} = d_p^{1,l} = 1$, if $l \leq m$, and $d_p^{j,l} = 0$, otherwise.

2) Let $p = -2m - 1$, $m \geq 0$. Then $d_p^{1,l} = d_p^{2,l} = 1$, if $l \leq m$, and $d_p^{j,l} = 0$, otherwise.

3) Let $p = -2m + 1$, $m \leq 0$. Then $d_p^{0,l} = d_p^{1,l} = 1$, if $l \geq m$, and $d_p^{j,l} = 0$, otherwise.

4) Let $p = -2m + 2$, $m \leq 0$. Then $d_p^{0,l} = d_p^{-1,l} = 1$, if $l \geq m$, and $d_p^{j,l} = 0$, otherwise.

References

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2. Frenkel, E.: Determinant formulas for the free field representations of the Virasoro and Kac-Moody algebras. Harvard Preprint, February 1992. Submitted to Phys. Lett. B

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