Erratum

Proof of Chiral Symmetry Breaking in Strongly Coupled Lattice Gauge Theory

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The “constant” C_n in Theorem 3.11 still has a m -dependence; Theorem 3.11 has to be restated as:

For all $m \in \mathcal{W}$ there is a $\kappa(m) > 0$ such that for all $n \in \mathbb{N}$ and $x_1, \dots, x_n \in \mathbb{Z}^v$,

$$|\langle \sigma^L \rangle^T| \leq \tilde{C}_n B_n(m) e^{-\kappa(m) \mathfrak{g}(x_1, \dots, x_n)}, \quad (1)$$

where $\tilde{C}_n$ depends only on n and m_0 (larger than the inverse radius of convergence of the cluster expansion), and

$$B_n(m) = \begin{cases} \max \{1, |\operatorname{Re} m|^{-n}\} & \text{for } |m| \leq m_0, \\ 1 & \text{for } |m| > m_0. \end{cases} \quad (2)$$

The reason for this is an error in our original proof: the bound $|\langle \sigma^L \rangle| \leq 1$ which we used holds only for real m . For general $m \in \mathcal{W}$ with $\operatorname{Re} m \neq 0$, the monomer-dimer results of Gruber and Kunz [1] imply only the weaker bound

$$|\langle \sigma^L \rangle| \leq |\operatorname{Re} m|^{-|L|}. \quad (3)$$

To prove clustering from this, fix $m' > 0$ and change the definition of u_L to

$$u_L(m) = \frac{1}{\mathfrak{g}(L)} \log \left(\frac{|\langle \sigma^L \rangle^T|}{\tilde{C}(L) \max \{1, m'^{-|L|}\}} \right), \quad (4)$$

then $u_L(m) \leq 0$ for all m with $|\operatorname{Re} m| > m'$, $u_L(m) < 0$ for $|m| > m_0$, and u_L is still subharmonic in m , so the Penrose-Lebowitz subharmonicity argument [2] implies $u(m) = \limsup u_L(m) < 0$ for all m with $|\operatorname{Re} m| > m'$, as worked out in our paper. Given m with $\operatorname{Re} m \neq 0$, Theorem 3.11, as stated above, is then obtained by taking e.g. $m' = \frac{1}{2} |\operatorname{Re} m|$. The proof also implies that $\kappa(m)$ is bounded below as $m \rightarrow 0$, $\kappa(m) \geq M |\operatorname{Re} m|$ for $|m| \leq m_0$, where M depends only on m_0 .

For real $m \neq 0$ and the two-point function we can get rid of $B_2(m)$:

$$|\langle \sigma_0 \sigma_x \rangle - \langle \sigma_0 \rangle \langle \sigma_x \rangle| \leq e^{-\kappa(m)|x|}, \quad (5)$$

where $|x| = \max_{i=1, \dots, v} \{|x_i|\}$. This can be seen as follows: let $m \in \mathbb{R}$. Denoting $\mathcal{L}_+ = \{L \in \mathcal{L} : \text{supp } L \subset \{x \in \mathbb{Z}^v : x_1 \geq 0\}\}$, our physical Hilbert space is the space $\mathcal{H}$ of equivalence classes of sequences in $l^1(\mathcal{L}_+)$, with scalar product (cf. Definition 3.14)

$$\langle \psi, \chi \rangle = \sum_{L, L' \in \mathcal{L}_+} \bar{a}_L b_{L'} \langle \sigma^{L+L'} \rangle \quad (6)$$

for ψ, χ represented by sequences $(a_L)_{L \in \mathcal{L}}$ and $(b_L)_{L \in \mathcal{L}}$. Let $\mathcal{D}$ be the subspace formed by the sequences with only finitely many nonzero entries. We denote the transfer operator corresponding to translations by $2e_1$ by T_1 , then $0 \leq T_1 \leq 1$ (second inequality from the boundedness at real m), and $T_1^* = T_1$, so the spectrum $\sigma(T_1) \subset [0, 1]$. By translation invariance, Ω , represented by $a_0 = 1, a_L = 0$ for $|L| > 0$, is an eigenvector of T_1 with eigenvalue 1. We denote the projection operator on Ω by P_Ω and also the spectral projection of T_1 on $I \subset [0, 1]$ by P_I .

By a similar argument as given above, one can show that for all $L, M \in \mathcal{L}$,

$$|\langle \sigma^{L+M} \rangle - \langle \sigma^L \rangle \langle \sigma^M \rangle| \leq C(m, L, M) e^{-\kappa(m)d(L, M)}, \quad (7)$$

where $d(L, M)$ is the minimal distance between the supports of L and M , the function $B_n(m)$ enters in the prefactor $C(m, L, M)$ and $\kappa(m)$ does not depend on L and M . For all $\psi, \chi \in \mathcal{D}$ and for all $t \in \mathbb{N}$,

$$|\langle \psi, T_1^t \chi \rangle| \leq \tilde{C}(m, \psi, \chi) e^{-\kappa(m)t}, \quad (8)$$

which implies that for all $f \in \mathcal{D} : P_{(e^{-\kappa}, 1)} f = 0$. Since $\mathcal{D}$ is dense in $\mathcal{H}$, $\sigma(T_1) \setminus \{1\} \subset [0, e^{-\kappa}]$. Let $x_i \in 2\mathbb{N}$, then

$$|\langle \sigma_0 \sigma_x \rangle - \langle \sigma_0 \rangle \langle \sigma_x \rangle| = \left| \left\langle \eta, (T_1^{x_1} - P_\Omega) \prod_{i=2}^v T_1^{x_i} \eta \right\rangle \right| \leq \|\eta\|^2 e^{-\kappa(m)x_1} \quad (9)$$

(η is the state corresponding to σ_0 and $T_i, i \geq 2$ are the unitary translations in spatial directions). Using the discrete rotational symmetry, we can repeat this argument for all other directions and get the bound. If x_i is not even for some i , the Schwarz inequality has to be applied once before proceeding with the given argument.

So, all our other results remain unchanged, because we only need clustering of the truncated two-point function in the form (5) for our proofs. Actually, for the lower bound for the chiral condensate the prefactor $B_n(m)$ does not play any role because we need clustering only at fixed $m > 0$ to identify the constant c in the formula for the Fourier-transformed two-point-function $\hat{T} = c\delta + \hat{g}$ as $c \langle \bar{\psi} \psi(x) \rangle^2$ and because the infrared bound for $\hat{g}$ is independent of the considerations about clustering. Only for the proof that the mass gap must go to zero as m vanishes, Remark 3.12 and the bound (4.19), we had to show that $B_2(m)$ is absent for real m .

References

1. Gruber, C., Kunz, H.: Commun. Math. Phys. **22**, 133 (1972)
2. Lebowitz, J., Penrose, O.: Commun. Math. Phys. **39**, 165 (1974)