ERRATUM

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Contrary to our claim in Theorem 4.1 the equation

$$\int_0^\varepsilon r(\psi(\sigma, x, r), \varepsilon) d\sigma = \varepsilon s(x, \varepsilon)$$

cannot be solved for r , in general. The correct version is

THEOREM 4.1. *For every function $s(x, \varepsilon)$ of class C^{k+p+1} there are functions $r(x, \varepsilon)$ and $T(x, \varepsilon)$ of class C^{k+p+1} such that*

$$t = U(\varepsilon, x) = \int_0^\varepsilon r(\psi(\sigma, x, r), \varepsilon) d\sigma = \varepsilon[s(x, \varepsilon) + T(x, \varepsilon)]$$

holds. $T(x, \varepsilon)$ is of the form $T(x, \varepsilon) = \varepsilon^p Q(x, \varepsilon)$ where $Q(x, \varepsilon)$ is of class C^{k+1} .

In other words: The transformation (4.2) takes the time- ε map of Eq. (4.1) to the time- $\varepsilon(s(x, \varepsilon) + T(x, \varepsilon))$ -map of Eq. (4.3).

PROOF. For given $r^q(x, \varepsilon)$ let $\psi^q(\tau, x, \varepsilon)$ be the solution of $dx/d\tau = r^q(x, \varepsilon)f(x)$ and put

$$(*)_q \quad T^q(x, \varepsilon) := \frac{1}{\varepsilon} \int_0^\varepsilon r^q(\psi^q(\sigma, x, \varepsilon), \varepsilon) d\sigma - s(x, \varepsilon).$$

Define recursively $r^0(x, \varepsilon) := 0$ and $r^{q+1}(x, \varepsilon) := r^q(x, \varepsilon) - T^q(x, \varepsilon)$. Let $A(q)$ be the following assertion:

- i) $r^q(x, \varepsilon)$ and $T^q(x, \varepsilon)$ are of class C^{k+p+1} .
- ii) $T^q(x, \varepsilon)$ is of the form $T^q(x, \varepsilon) = \varepsilon^q Q^q(x, \varepsilon)$ where $Q^q(x, \varepsilon)$ is of class $C^{k+p+1-q}$.

Clearly $A(0)$ holds. We show that $A(q-1)$ implies $A(q)$ for $q = 1, 2, \dots, p$. Let $A(q-1)$ hold for $q-1 < p$. We show $A(q)$:

- i) $r^q(x, \varepsilon) := r^{q-1}(x, \varepsilon) - T^{q-1}(x, \varepsilon)$ is of class C^{k+p+1} . Therefore, the solutions $\psi^{q-1}(\tau, x, \varepsilon)$ and $\psi^q(\tau, x, \varepsilon)$ are of class C^{k+p+1} . It follows from Eq. $(*)_q$ that $T^q(x, \varepsilon)$ is of class C^{k+p+1} . Note that one derivative is gained by integrating with respect to ε and that the extra derivative is lost by dividing the integral by ε .
- ii) Taking the difference of Eqs. $(*)_q$ and $(*)_{q-1}$ we obtain

$$Q^q(x, \varepsilon) = \frac{1}{\varepsilon^2} \int_0^\varepsilon \frac{r^{q-1}(\psi^q(\sigma, x, \varepsilon), \varepsilon) - r^{q-1}(\psi^{q-1}(\sigma, x, \varepsilon), \varepsilon)}{\varepsilon^{q-1}} d\sigma \\ - \frac{1}{\varepsilon^2} \int_0^\varepsilon [Q^{q-1}(\psi^q(\sigma, x, \varepsilon), \varepsilon) - Q^{q-1}(x, \varepsilon)] d\sigma.$$

By induction the solutions ψ^q and ψ^{q-1} satisfy differential equations which differ only by terms of order ε^{q-1} . Therefore, we have $\psi^q(\sigma, x, \varepsilon) - \psi^{q-1}(\sigma, x, \varepsilon) = \sigma \varepsilon^{q-1} \delta(\sigma, x, \varepsilon)$ where δ is of class $C^{k+p+1-(q-1)}$. It follows that the integrand of the first integral is of class $C^{k+p+1-(q-1)}$ and has the form $\sigma \Delta(\sigma, x, \varepsilon)$. The same assertion holds for the integrand of the second integral. It now follows that $Q^q(x, \varepsilon)$ is of class $C^{k+p+1-q}$. (One derivative is gained by integrating and two derivatives are lost when dividing by ε^2). ■

The result of Theorem 4.1 is used as follows in the proof of Theorem 5.2. First take $s(x, \varepsilon)$ according to Theorem 3.1 and then choose $r(x, \varepsilon)$ and $T(x, \varepsilon)$ as in Theorem 4.1. Then the integration method Φ with variable step size $h = \varepsilon s(x, \varepsilon)$ is an $O(\varepsilon^{p+1})$ C^{k+1} -approximation to the "Poincaré map" $\varphi(\varepsilon(s(x, \varepsilon) + T(x, \varepsilon)), x)$ of Eq. (4.3). Note that the C^{k+1} -transformation (4.2) takes $\varphi(\varepsilon(s(x, \varepsilon) + T(x, \varepsilon)), x)$ to the Poincaré map $\psi(\varepsilon, x)$ of Eq. (4.1). Hence, by means of the transformation (4.2) the method Φ may be interpreted as an integration method $\tilde{\Phi}$ of Eq. (4.1) with constant step size ε . $\tilde{\Phi}(\varepsilon, x)$ is $O(\varepsilon^{p+1})$ C^{k+1} -close to $\psi(\varepsilon, x)$.