Book Reviews

Bergström, H.: Weak Convergence of Measures. Academic Press 245 p., US-\$ 49.50.

The contents of this book split into two parts of various nature. The first three chapters develop a rather general theory of measure and integral on normal σ -topological spaces; the last three chapters are concerned with weak convergence on the classical metric spaces of probability theory. One aim of the author is it to incorporate the approach to measure theory taken by A.D. Alexandrov (Additive set functions in abstract spaces I–III). So measures are not necessarily σ -additive (or ' σ -smooth'); and to some extent topological spaces are replaced by A (lexandrov)-spaces (only countable unions of open sets are open again).

Let us sketch the contents of the book. Chapter I (Spaces, Mappings, and Measures) assembles basic notions and concepts. In Chapter II (Integrals, Bounded Linear Functionals, and Measures) the connection between measures and functionals is established. The core is Theorem II.4.1 (entitled 'Alexandrov's first theorem') on the representation of certain functionals by means of appropriate measures. The central part of the book is Chapter III (Weak Convergence in Normal Spaces). Here one finds 'Alexandrov's second theorem' (Theorem III.1.1) on the characterization of weak convergence of measures on Stone vector lattices. Moreover there is the celebrated 'Alexandrov's third theorem' (Theorem III.4.1) saying that sequences of σ -smooth measures can converge weakly to σ -smooth measures only.

Chapter IV (Weak Convergence on $\mathbb{R}^{(k)}$) contains a discussion of classical limit theorems (for convolution products) and some results on infinitely divisible probability measures over Euclidean spaces. Here the author strongly applies his own techniques. This chapter covers a good part of the author's earlier book 'Limit Theorems for Convolutions'. In Chapters V (Weak Convergence on the C- and D-Spaces) and VI (Weak Convergence in Separable Hilbert Spaces) the author applies a 'reduction principle' in order to solve limit problems for measures on infinite-dimensional spaces by reducing them to weak limit problems on appropriate finite-dimensional spaces.

With this book the author offers an approach to weak convergence of measures which is essentially different from other treatments. For specialists it will be useful to have at hand a presentation which is original in selection and techniques. Moreover the author assembles some relevant results which are difficult to be found elsewhere.

On the other hand beginners in the field might have some problems in reading this book. The general setup of its first part (no σ -additivity, σ -topological spaces) is not justified by the second part where only σ -smooth measures on (complete) metric spaces come into consideration. Moreover the author sometimes deviates from standard notions and notations. So what is termed ' σ -compact' by him (cf. p. 5) is usually called 'countably compact'. $\hat{\mu}$ denotes the distribution function of a probability measure μ (p. 103) whereas $\tilde{\mu}$ denotes its Fourier transform (p. 108). There are also numerous missprints and omissions. For example in the formulation of Theorem I.1.12 (p. 33) one has to substitute μ by $\underline{\mu}$ in four places. In Theorem II.2.10 (p. 69) the condition of absolute continuity is missing. The definition of a monotone class (p. 3) is incomplete; the proof of Theorem I.1.3 (p. 4) is incorrect. The bibliography contains gaps; for example there is no reference to the work of F. Topsøe on weak convergence.

E. Siebert, Tübingen