CORRECTION

A Note On Necessary Best Estimator Of Order Two

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Equations (14) to (18) are incorrect. The corrected equation (14) is as follows:

$$E\left(\frac{1}{\pi_{\lambda} \pi_{\lambda'}}\right) = \sum_{s \supset \lambda \lambda'} \frac{p(s)}{\pi_{\lambda} \pi_{\lambda'}} \\ = \frac{\pi_{\lambda \lambda'}}{\pi_{\lambda} \pi_{\lambda'}} \quad (14)$$

On the same lines, let

$$E\left(\frac{e_{s\lambda}}{\pi_{\lambda'}}\right) = E_1 \quad (15)$$

$$E \frac{e_{s\lambda'}}{\pi_{\lambda}} = E_2 \quad (16)$$

Now equation (17) changes to

$$E(\beta_{s\lambda} \beta_{s\lambda'}) = \frac{\pi_{\lambda \lambda'}}{\pi_{\lambda} \pi_{\lambda'}} + E_1 + E_2 + E(e_{s\lambda} e_{s\lambda'}) \quad (17)$$

Subsequently equation (18) changes to

$$Q' = \text{Var}(t) - \text{Var}(t_{HT}) \\ = \sum_{\lambda=1}^N E(e_{s\lambda}^2) Y_{\lambda}^2 \\ + \sum_{\lambda \neq \lambda'=1}^N [E_1 + E_2 + E(e_{s\lambda} e_{s\lambda'})] Y_{\lambda} Y_{\lambda'} \quad (18)$$

The Horvitz and Thompson estimator is the necessary best estimator of order two if for $\lambda \neq \lambda'$

$$E(e_{s\lambda}^2) E(e_{s\lambda'}^2) - [E_1 + E_2 + E(e_{s\lambda} e_{s\lambda'})]^2$$

is non-negative.

Consider the restricted class of unbiased estimators where for all s ,

$$\beta_{s\lambda} = \begin{cases} \beta_{s\lambda} & \lambda \neq \lambda' = 1, 2, \dots, N. \\ \frac{1}{\pi_{\lambda'}} & \text{for } \lambda = \lambda' \end{cases}$$

For these estimators $e_{s\lambda} = 0$ for all s , and the above expression reduces to

$$-E_1^2$$

which is negative.

This establishes the result that the necessary best estimator of order two does not exist in Lu.