A CORRECTION OF "THE NILSTUFE OF THE DIRECT SUM OF RANK 1 TORSION FREE GROUPS"

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Professor R. Beaumont has pointed out an error in the proof of [1, theorem 1] which in turn gave rise to an error in the proof of theorem 3. He also provided the following counter-example to theorem 3: Let G_1 be a rank one torsion free group of type $(1, \infty, 1, \infty, \dots)$ and let G_2 be a rank one torsion free group of type $(0, 1, 0, 1, \dots)$. G_1 and G_2 are nil groups [2, theorem 121.1], but it is easy to show that $N(G_1 \oplus G_2) = \infty$.

In what follows a theorem concerning homogeneous groups will be proved, as a consequence of which a weakened version of theorem 3 will be obtained.

THEOREM. *Let $G = G_1 \oplus G_2$, G_i a homogeneous group, $1 \leq i \leq 2$, with $T(G_1) > T(G_2)$, $T(G_2)$ not idempotent, and $v(G_1) = n$. Then $v(G) \leq 2n + 1$.*

PROOF. Let $g \in G_1$, $h \in G$. For every multiplication on G , $T(gh) \geq T(g) \geq T(G_1) > T(G_2)$. Let $gh = g_1 + g_2$, $g_i \in G_i$, $1 \leq i \leq 2$. If $g_2 \neq 0$, then $T(gh) = T(g_2) = T(G_2)$. A contradiction. Hence:

A) $G_1 \cdot G \subseteq G_1$ and similarly

B) $G \cdot G_1 \subseteq G_1$

for every multiplication on G .

Let $g, h \in G_2$. For every multiplication on G , $T(gh) \geq 2T(G_2) > T(G_2)$ since $T(G_2)$ is not idempotent. As above, this inequality yields:

C) $G_2 \cdot G_2 \subseteq G_1$

for every multiplication on G .

A), B), and C) imply

D) $G \cdot G \subseteq G_1$ for every associative multiplication on G .

For every multiplication on G , $G^{2n+2} \subseteq (G_1)^{n+1}$ by D). $(G_1)^{n+1} = 0$ by virtue of the fact that $v(G_1) = n$.

COROLLARY. *Let $G = G_1 \oplus G_2$, G_i a rank one torsion free group, $v(G_i) = 1$, $1 \leq i \leq 2$. If $T(G_1) > T(G_2)$ then $v(G) \leq 3$.*

References

- [1] S. FEIGELSTOCK, The nilstufe of the direct sum of rank 1 torsion free groups, *Acta Math. Acad. Sci. Hungar.*, **24** (1973), 269—272.
- [2] L. FUCHS, *Infinite Abelian Groups*, Vol. 2, Academic Press (New York and London, 1973).

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