Erratum

Function Weighted Measures and Orthogonal Polynomials on Julia Sets¹

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Theorem 3.4 in the above paper is not quite strong enough to give Corollary 3.1. However, the following strengthening of Theorem 3.4 does imply Corollary 3.1. (Note that $z \notin J$ in the hypothesis of Corollary 3.1 should read $z \notin I$.)

Theorem 3.5. *Let c, Δ, γ , and φ be as in (3.20) and (3.21) in the above paper. Let*

$$f = \max_s \|p_s\|_\infty, \quad s = 0, 1, 2, \dots, d^2 - 1.$$

Then

$$(*) \quad \|p_{md^n+s}\|_\infty \leq f\varphi^{n-1}, \quad n = 1, 2, \dots, \quad s = 0, 1, \dots, d^n, \quad m = 1, 2, \dots, d-1.$$

Proof. We prove the result by induction since $\|p_{d^2}\|_\infty = \|p_d(T(x))\|_\infty = \|p_d(x)\|_\infty$ the result is true for $n = 1$. Assume now that $(*)$ is true for $n = k-1$. If $s = ld^i$, $l < d$, and $0 < i \leq k$, then

$$\begin{aligned} \|p_{md^k+s}\|_\infty &= \|p_{md^k+ld^i}\|_\infty \\ &= \|p_{md^{k-1}+ld^{i-1}}(T(x))\|_\infty \\ &= \|p_{md^{k-1}+ld^{i-1}}(x)\|_\infty \leq f\varphi^{k-2} \leq f\varphi^{k-1} \end{aligned}$$

since $\varphi \geq 1$. Write $s = ld^i + j$, $0 < i < k$, $j = 1, 2, \dots, d-1$, then from (3.18), (3.21), and (3.23) of the above paper we find

$$\begin{aligned} \|p_{d(md^{k-1}+ld^{i-1}+j)}\|_\infty &\leq \frac{c}{\gamma} (\Delta \|p_{md^{k-1}+ld^{i-1}}\|_\infty + \|p_{md^{k-1}+ld^{i-1}+1}\|_\infty) \\ &\leq \frac{c}{\gamma} (\Delta + 1) f\varphi^{k-2} = f\varphi^{k-1}, \end{aligned}$$

which gives the result. ■

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¹ D. Bessis, J. S. Geronimo, and P. Moussa (1988): *Function weighted measures and orthogonal polynomials on Julia sets*. Constr. Approx., 4:157–173.

Remark. Using the fact that $\prod_{i=1}^n a(nd+i) = a(n+1)$ and (3.18) modified so that it works for the orthonormal polynomials, i.e.,

$$\hat{p}_{nd+j} = \frac{1}{w(x)} \left\{ \frac{a(n+1)}{a(nd+j+1)} \hat{p}_{d-j-1}^{(nd+j+1)}(x) \hat{p}_{nd}(x) + \frac{a(n+1)}{a(nd+1)} \hat{p}_{d-1}^{(nd+1)}(x) \hat{p}_{(n+1)d}(x) \right\},$$

and using techniques similar to those above it is not hard to show that the orthonormal polynomials exhibit regular asymptotic behavior [1].

Reference

1. H. STAHL, V. TOTIK (1990): *Nth root asymptotic behavior of orthonormal polynomials*. In: *Orthogonal Polynomials: Theory and Practice* (P. Nevai, ed.). NATO ASI Series. Dordrecht: Kuwer, pp. 395-417.