Convexity-like inequalities for averages in a convex set

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For a function $f: X \rightarrow R \cup \{+\infty\}$, convex and finite over an algebraically open convex subset C of a linear space X , we provide a simple necessary and sufficient condition for the "convexity inequality"

$$f\left(\sum \lambda_i x_i\right) \leq \sum \lambda_i f(x_i) \quad (C_n)$$

to be satisfied for every integer $n \geq 2$ and all $x_1, \dots, x_n \in X$ and $\lambda_1, \dots, \lambda_n \in R_+$ such that $\sum \lambda_i = 1$ and

$$\sum \lambda_i x_i \in C$$

(i.e. if *just the average* $\sum \lambda_i x_i$ lies in the set C of convexity, *not* the points $x_1, \dots, x_n$ themselves).

Our condition is simply that f majorizes a certain convex function, depending only on the restriction f_C of f to C , namely the *smallest convex extension* of f_C , which is given explicitly.

We apply this result to *enlarge* the set of n -tuples $(x_1, \dots, x_n) \in X^n$ satisfying (C_n) , beyond C^n , the domain ensured by the convexity of f on C . A number of examples show that such an *extension of the domain of validity* of (C_n) can be easily obtained when $X = R$.

Sur les solutions globales de l'équation des cocycles

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Let $\Phi: \mathbb{R}^+ \times X \mapsto X$ be a semidynamical system and $f: \mathbb{R}^+ \times X \mapsto \mathbb{R}^+$ be a cocycle of (X, Φ) , i.e. f verifies the following functional equation:

$$\forall x \in X; s, t \geq 0: \quad f(s+t, x) = f(t, x)f(s, \Phi(t, x)). \quad (E)$$

Under convenient conditions, we give an explicit form of the globally measurable or continuous solutions of (E) and we study the particular case when $X = \mathbb{R}$.