Strong products of χ -critical graphs

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Let $G[H]$ be the lexicographic product and let $G \boxtimes H$ be the strong product of the graphs G and H . It is proved that, if G is a χ -critical graph, then, for any graph H ,

$$\chi(G[H]) \leq \chi(H)(\chi(G) - 1) + \left\lceil \frac{\chi(H)}{\alpha(G)} \right\rceil.$$

This upper bound is used to calculate several chromatic numbers of strong products. It is shown in particular that for $k \geq 2$, $\chi(C_5 \boxtimes C_5 \boxtimes C_{2k+1}) = 10 + \lceil 5/k \rceil$, and for $k \geq 2$ and $n \geq 1$, $\chi(\overline{C_{2k+1}} \boxtimes K_n) = kn + \lceil n/2 \rceil$. That the general upper bound cannot be improved for graphs which are not χ -critical is demonstrated by two infinite series of graphs. The paper is concluded with an application to graph retracts: if for some graph H with at least one edge $\chi(G[H]) = \chi(G)\chi(H)$, then no χ -critical subgraph G' of G , $G' \neq K_n$, is a retract of G .

On two conditional forms of the equation $\|f(x+y)\| = \|f(x) + f(y)\|$

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With reference to the functional equation

$$\|f(x+y)\| = \|f(x) + f(y)\|$$

for $f: X \rightarrow E$, where X and E are real linear spaces and $E = (E, \|\cdot\|)$ is normed, we consider the following conditional equations

- (a) $(x, y) \in X \times X, \quad f(x+y) \neq 0 \Rightarrow \|f(x+y)\| = \|f(x) + f(y)\|$
- (b) $(x, y) \in X \times X, \quad f(x) + f(y) \neq 0 \Rightarrow \|f(x+y)\| = \|f(x) + f(y)\|,$

corresponding to the known Mikusinski and Dhombres conditional forms of the Cauchy equation $f(x+y) = f(x) + f(y)$, respectively.

In this paper, without any regularity conditions on f , we shall prove that each of (a) and (b) is equivalent to the Cauchy equation, provided that $(E, \|\cdot\|)$ is a strictly normed real space.