ERRATUM AND ADDENDUM: QUANTIZATION OF THE MAXWELL FIELD IN LINEAR CONFORMAL INVARIANT GAUGES

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The argument used in the proof of Theorem 3.1 for the elimination of $G^-(x)$, $\Delta^-(x)$ in the general expression for $h(x)$ is incorrect [1]: actually, the non-degeneracy of $\langle \cdot, \cdot \rangle$ implies only $A(f)\psi_0 = 0$ when the Fourier transforms $\hat{f}_\mu$ of the components of the test-function have their supports in the domain $k_0 < 0$ and verify $k^\mu \hat{f}_\mu(k) = 0$.

Therefore, we must cope with the following question: Is there a quasi-free field theory for which the two-points function contains a linear combination of $G^-(x)$ and $\Delta^-(x)$? As we shall show now, the answer is affirmative.

Indeed, let H_- be the Hilbert space of two components functions $\omega_i(\vec{k})$, $i = 1, 2$, such that:

$$\int \frac{d^3 k}{|\vec{k}|} \left(\frac{|\vec{k}|}{1 + |\vec{k}|^2} |\omega_1(\vec{k})|^2 + \frac{1 + |\vec{k}|^2}{|\vec{k}|^2} |\omega_2(\vec{k})|^2 + \sum_{i=1}^3 \left| \frac{\partial \omega_2}{\partial k_i}(\vec{k}) \right|^2 \right) < \infty.$$

We define in H_- an indecomposable representation $U_-(a, \Lambda)$ of the Poincaré group $\mathcal{P}$ by $(k_0 = |\vec{k}|)$:

$$U_-(a, \Lambda)\omega_1(\vec{k}) = e^{-ia \cdot k} \left(\omega_1(\Lambda^{-1}k) + \frac{1}{|\vec{k}|} \left(\Lambda_i^0 \frac{\partial \omega_2}{\partial k_i}(\Lambda^{-1}k) - ia_0 \omega_2(\Lambda^{-1}k) \right) \right),$$

$$U_-(a, \Lambda)\omega_2(\vec{k}) = e^{-ia \cdot k} \omega_2(\Lambda^{-1}k).$$

Then, the Hermitian form:

$$B_-^m(\omega, \omega') = \int \frac{d^3 k}{|\vec{k}|} \left(\omega_1(\vec{k}) \overline{\omega'_2(\vec{k})} + \omega_2(\vec{k}) \overline{\omega'_1(\vec{k})} - \frac{\omega_2(\vec{k}) \overline{\omega'_2(\vec{k})}}{|\vec{k}|} \right) - m_- \int \frac{d^3 k}{|\vec{k}|} \omega_2(\vec{k}) \overline{\omega'_2(\vec{k})}$$

is, up to a factor, the general expression of an $U_-(a, \Lambda)$ -invariant non-degenerate Hermitian form. Furthermore, for any test-function f , the operator defined by:

$$(\Pi_- f)_1(\vec{k}) = \frac{1}{|\vec{k}|} \frac{\partial \Omega}{\partial k_0}(-|\vec{k}|, -\vec{k})$$

$$(\Pi_- f)_2(\vec{k}) = \Omega(-|\vec{k}|, -\vec{k})$$

where $\Omega(k) = k^\mu f_\mu(k)$, is an intertwining operator between $U_-(a, \Lambda)$ and the canonical representation of $\mathcal{P}$ in the test function space.

Let now $\mathcal{H}_-$ be the symmetrical Fock space built on H_- . Let $\mathcal{U}_-(a, \Lambda)$ and $\mathcal{B}_-(\cdot, \cdot)$ the representation of $\mathcal{P}$ and the invariant Hermitian form induced by $U_-(a, \Lambda)$ and $d_- B_-^{M-1/d-}(\omega, \omega)$, respectively. We define on $\mathcal{H}_-$, the creation operators $b_i^+(\vec{k})$, $i = 1, 2$, and the annihilation operators $b_i(\vec{k})$ by adjunction with respect to $\mathcal{B}_-(\cdot, \cdot)$. Given the test-function f , let us introduce on $\mathcal{H}_-$ the following covariant operator distribution:

$$A_-(f) = b^+(\Pi_- f) + \overline{b(\Pi_- f)}.$$

The two-points function corresponding to this operator is equal exactly to the negative frequency part of $G_{\alpha\beta}(x)$.

Let now $\mathcal{H}_+$ be the Fock space associated with the generalized Lorentz-gauge indexed by d_+ and let $A_+(f)$ the corresponding vector-potential quantum field (see Ref. 1).

Let us introduce in $\mathcal{H} = \mathcal{H}_+ \otimes \mathcal{H}_-$, the following covariant operator distribution:

$$A(f) = A_+(f) \otimes I_- + I_+ \otimes A_-(f)$$

where $I_\pm$ are the identity operators of $\mathcal{H}_\pm$. Then, the two-points function corresponding to this field turns out to be precisely that one implied by the general expression for $h(x)$.

A direct consequence of this construction is that the Theorem 3.1 itself is no longer valid, at least in the framework we adopted for it.

Nevertheless, the introduction of the space $\mathcal{H}_-$ appears clearly as artificial and spurious, and it is natural to look for an additional postulate which gets rid of it. This can be done in the following way:

Indecomposability postulate:

The representation of $\mathcal{P}$ restricted to the one-particle space must be an indecomposable representation.

This condition implies that the representation $U_-(a, \Lambda)$ does not appear, at least if we look for a non-trivial theory. But then $d_- = M_- = 0$ in the general expression for $h(x)$, so that we can state again Theorem 3.1 in this slightly restricted frame.

REFERENCE

1. Rideau, G., 'Quantization of the Maxwell Field in Linear Conformal Invariant Gauges', *Letters Math. Phys.* 3, 229 (1979).

(Received May 8, 1979)