A Correction to the Paper « An Existence Theorem for Compressible Viscous Fluids » (*).

ALBERTO VALLI (Povo, Trento)

In [2], the proof of the convergence of the successive approximations is not complete. In fact, we have proved the existence of a subsequence $(v_{n_k}, \theta_{n_k}, \varrho_{n_k})$ which converges, but we cannot pass to the limit in (2.4), (2.5) and (2.6), without proving that $(v_{n_k-1}, \theta_{n_k-1}, \varrho_{n_k-1})$ also converges to the same limit.

This result does not seem easy to be proved, hence we prefer to utilize a slightly different approach. Actually, we complete the proof of the existence of a solution by a fixed point argument.

Define Z the map $(v_{n-1}, \theta_{n-1}, \varrho_{n-1}) \rightarrow (v_n, \theta_n, \varrho_n)$, where v_n is obtained in (2.4), θ_n in (2.5) and ϱ_n in (2.6). Set

$$R_T \equiv \left\{ (v, \theta, \varrho) \mid \|v\|_{4,2,T} \leq C'_1 F, \|\theta\|_{4,2,T} \leq C'_6 F, \|\varrho\|_{\infty,3} \leq F, \|\dot{\varrho}\|_{\infty,2} \leq C_{11}, v(0) = v_0, \right. \\ \dot{v}(0) = -\tilde{A}v_0 + b(0) - (v_0 \cdot \nabla)v_0 - \frac{1}{\varrho_0} \nabla p_0, \theta(0) = \theta_0, \dot{\theta}(0) = -\tilde{B}\theta_0 + \frac{r(0)}{c_0} - v_0 \cdot \nabla \theta_0 - \\ \left. - \frac{1}{\varrho_0 c_0} \theta_0 \frac{\partial p_0}{\partial \theta} \operatorname{div} v_0 + \frac{\mu_0}{2\varrho_0 c_0} \sum_{i,k} (D_k v_0^i + D_i v_0^k)^2 + \frac{1}{\varrho_0 c_0} \left(\zeta_0 - \frac{2}{3} \mu_0 \right) (\operatorname{div} v_0)^2, \right. \\ \left. \varrho(0) = \varrho_0, \bar{\varrho}_1 \leq \varrho(t, x) \leq \bar{\varrho}_2 \text{ in } \bar{Q}_T \right\},$$

where C_{11} is a constant (which depends only on the data of the problem and not on T) such that

$$\|v\|_{4,2,T} \leq C F'_1, \quad \|\varrho\|_{\infty,3} \leq F, \quad v(0) = v_0, \quad \dot{v}(0) = -\tilde{A}v_0 + b(0) - (v_0 \cdot \nabla)v_0 - \frac{1}{\varrho_0} \nabla p_0$$

implies that $\|v \cdot \nabla \varrho + \varrho \operatorname{div} v\|_{\infty,2} \leq C_{11}$.

The calculations performed in [2] show that there exists $T_1 \in]0, T_0]$ such that $Z(R_{T_1}) \subset R_{T_1}$. Moreover, R_{T_1} is convex, and from classical compactness theorems (see [2], pag. 206 for the details) it is compact in

$$X \equiv \{(v, \theta, \varrho) \mid v, \theta, \varrho \in C^0([0, T_1]; H^2(\Omega))\}.$$

(*) Entrata in Redazione il 24 settembre 1982.

Finally, one proves easily, by an energy argument, that if $(v_k^*, \theta_k^*, \varrho_k^*) \in R_{T_1}$ and $(v_k^*, \theta_k^*, \varrho_k^*) \xrightarrow{k} (v^*, \theta^*, \varrho^*)$ in X , then $Z(v_k^*, \theta_k^*, \varrho_k^*) \xrightarrow{k} Z(v^*, \theta^*, \varrho^*)$ in $C^0([0, T_1]; L^2(\Omega))$, and consequently in X , as R_{T_1} is compact in X .

Hence by Schauder's fixed point theorem one finds a fixed point of Z , that is a solution of our problem. Furthermore, from the uniqueness theorem (see [3]) the fixed point is unique.

For more details about this procedure we refer to [1], where a different problem is studied by a similar fixed point argument.

REFERENCES

- [1] P. SECCHI - A. VALLI, *A free boundary problem for compressible viscous fluids*, J. Reine Angew. Math., to appear.
 - [2] A. VALLI, *An existence theorem for compressible viscous fluids*, Ann. Mat. Pura Appl., **130** (1982), pp. 197-213.
 - [3] A. VALLI, *Uniqueness theorems for compressible viscous fluids, especially when the Stokes relation holds*, Boll. Un. Mat. It., Anal. Funz. Appl., **18-C** (1981), pp. 317-325.
-