Correction to «Periodic Solutions of a Forced Liénard Equation» (*).

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1. A missing hypothesis.

In the paper of title [1], under continuity conditions we considered the equation

$$(1) \quad x'' + f(t, x, x')x' + g(x) = e(t, x, x')$$

where it is assumed that

$$(2) \quad \text{there is an } \alpha > 0 \text{ with } f(t, x, y) \geq \alpha \text{ and } e(t, x, y) \text{ bounded,}$$

$$(3) \quad |g(x)| > |e(t, x, y)| \text{ for all large } |x|,$$

$$(4) \quad g(x)/x \rightarrow 0 \quad \text{as } |x| \rightarrow \infty,$$

and there is a $T > 0$ with

$$(5) \quad e(t + T, x, y) = e(t, x, y) \quad \text{and} \quad f(t + T, x, y) = f(t, x, y).$$

We stated that (2)-(5) imply that (1) has a T -periodic solution.

Early in the proof (p. 343, line 7 from the bottom) we derived a necessary condition for a periodic solution and used it extensively throughout the remainder of the proof. We noted there that (1) has a periodic solution only if, for $h(s) = f(s, x(s), y(s))$, then

$$(7) \quad x'(t) = \int_{-\infty}^t \left(\exp - \int_u^t h(s) ds \right) [-g(x(u)) + e(u, x(u), y(u))] du$$

has zeros. Thus, a necessary condition is that the integrand changes sign along a periodic solution. Condition (3) ensured that if this were to happen then there must be a set in R^2 with x bounded in which this takes place.

Thus, we clearly needed to add the assumption that

$$(G) \quad g(x) = e(t, x, y) \quad \text{has a solution.}$$

Using (G) and (7) we deduced that there is a $g^* > 0$ such that for each T -periodic

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solution $x(t)$, there is a t_1 with

$$(8) \quad |x(t_1)| < g^*.$$

The proof then involves defining a homotopy equation

$$(12) \quad x(t) = \lambda \int_{-\infty}^t e^{-m(t-v)} \left\{ mx(v) - \int_{-\infty}^v D(v, u)[g(x(u)) - e(u, x(u), x'(u))] du \right\} dv,$$

where $D(t, u) = \exp - \int_u^t f(s, x(s), x'(s)) ds$ and $m > 0$, and finding an a priori bound on all possible T -periodic solutions of (12) for $0 < \lambda < 1$.

It is here that a major error occurred. We used the bound (8) for all possible T -periodic solutions of (12). But (8) was derived only for (1), not (12). Not only have all efforts to prove (8) for (12) failed, but consideration of piecewise continuous examples has convinced us that it is probably not true.

One of the major novelties of the paper was that there was no sign condition on g . But, in addition, a new Liapunov function was constructed and a homotopy argument was given involving a more general f and e than had previously been offered. Thus, there is some point in showing that (8) will hold when g satisfies a sign condition. We must replace (3) by

$$(3^*) \quad |g(x)| > |e(t, x, y)| \quad \text{and} \quad xg(x) > 0 \quad \text{for all } |x| > g^*.$$

The theorem should read:

THEOREM. – If (2), (3*), (4), and (5) hold, then (1) has a T -periodic solution.

The proof given will be complete if we show that (2), (3*), (4), and (5) imply that if x is a T -periodic solution of (12) for $0 < \lambda < 1$, then there is a t_1 with (8) holding.

If x changes sign, then (8) holds. Thus, we suppose that $x(t) > 0$ for all t ; a symmetric proof holds if $x(t) < 0$. Denote the double integral in (12) by $I(t)$. If the maximum of x occurs at $x(t_2)$, then

$$x(t_2) \leq x(t_2) \lambda \int_{-\infty}^{t_2} e^{-m(t_2-v)} m dv - I(t_2) = \lambda x(t_2) - I(t_2).$$

Since $\lambda < 1$, $I(t_2) < 0$. This means that $g - e$ is negative at some value t_1 . By (3*) this implies that (8) holds.

We thank Professor Gabriele Villari for pointing out that a hypothesis was missing and we sincerely regret any difficulties this has caused a reader.

REFERENCE

- [1] T. A. BURTON, *Periodic solutions of a forced Liénard equation*, Ann. Mat. Pura Appl., CLXVII (1994), pp. 341-350.