$\pi(\omega, \varphi) \in F\}$ under the continuous mapping Π , and a continuous image of a compact set is compact. We thus conclude that the set $(\pi(\omega))^{-1}(F)$ is not closed in Q , and hence the function $\pi(\omega, \cdot)$ cannot be continuous on Q . We have thus established that almost all the sample paths of the natural modification π are discontinuous functions on Q . It is now clear that Q is not metrizable, since otherwise π , by definition would be a natural modification. Q.E.D.

In conclusion, I would like to correct some errata in [1]. On p. 37, Theorem 2(c), read: "and the mapping Ξ is linear." On p. 41, end of Theorem 7: " $E|f - \Xi(t)| \leq \varepsilon$ " On p. 42, line 4 from bottom: "open subsets of zero measure." On p. 50, proof of Lemma 4: "we write that $\mathcal{L}[\omega]$ is defined and equal to α ." Moreover, the last sentence of the article, relating to the proof of Corollary 6, is evidently wrong. A correct proof of "Corollary 6" will be found in the article listed in [1] as reference [18].

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CORRIGENDUM AND ADDENDUM TO THE ARTICLE "HILBERT SUBSPACES OF FULL MEASURE IN ℓ_p SPACES

B. Diallo

The following corrigenda should be introduced in my article [1]. Theorem 1 (p. 4) should be restated as follows.

THEOREM 1. Every probability measure μ in ℓ_p space ($1 \leq p < 2$), in which linear continuous functionals are square summable and the topology of convergence in mean square on the adjoint space ℓ_q coincides with the topology of convergence in measure, is concentrated on some oriented Hilbert subspace.

The corresponding changes should be made throughout the article [1].

Before proving this theorem, note that gaussian measures have this property, and that for every discrete measure μ , with weights p_k at the points x_k there is a sequence λ_k such that the discrete measure $\tilde{\mu}$, with weights p_k at the points $\lambda_k x_k$ has the above property. The theorem thus holds, in particular, for all discrete and gaussian measures (and the subspace H_p introduced in [1] is of full measure). Moreover, if the conditions of the theorem hold for each of the measures $\{\mu_k, k=1, \dots\}$, the proposition is also true for an arbitrary convex combination of these measures $\mu = \sum_k p_k \mu_k$.

Proof of Theorem 1. Let $V \subset \ell_q$ be the trace on ℓ_q of the unit sphere in $L_2(\mu)$ and let $\mathcal{E} = V^\circ \subset \ell_p$ (in case of gaussian measure, $\mathcal{E}$ is a dispersion ellipsoid). In order to obtain the required result from the Minlos-Sazonov theorem, it suffices to prove that $\sum_k (\sup \{x_k :$

Translated from Zapiski Nauchnykh Seminarov Leningradskogo Otdeleniya Matematicheskogo Instituta im. V. A. Steklova AN SSSR, Vol. 72, pp. 212-214, 1977.

$x \in \mathcal{E})^p < \infty$, i.e., that the ellipsoid $\mathcal{E}$ is contained in some oriented parallelepiped with the vertex $\bar{x} = (\bar{x}_k) \in \ell_p$; then the sought subspace is the Hilbert space $H = \{x \in \ell_p : \sum x_k^2 / \bar{x}_k^{2-p} < \infty\} \subset \ell_p$, and the sum of the squares of the semiaxes of the ellipsoid $\mathcal{E}$ in the norm of H is not greater than the square of the norm in H of the element $\bar{x} \in H$. Thus the Minlos-Sazonov theorem can be applied, since the characteristic functional χ_μ of the measure μ is naturally continuous in the L_2 -norm.

Now let $\varepsilon_n \downarrow 0$. First assume that $\int \|x\|_{\ell_p}^p d\mu < \infty$ and define the numbers $\beta_k^{(n)}$ by the inequalities

$$\mu_{e_k}((- \beta_k^{(n)}, \beta_k^{(n)})) \leq 1 - \varepsilon_n \leq \mu_{e_k}([- \beta_k^{(n)}, \beta_k^{(n)}]),$$

where μ_f is the distribution of the values of the functional $f \in \ell_q$, e_k are the coordinate functionals. We will show that $\forall n \sum_k (\beta_k^{(n)})^p < \infty$. Indeed,

$$\int \|x\|_{\ell_p}^p d\mu = \sum \int |x_k|^p d\mu,$$

$$\int |x_k|^p d\mu = \int_{|x_k| < \beta_k^{(n)}} |x_k|^p d\mu + \int_{|x_k| \geq \beta_k^{(n)}} |x_k|^p d\mu > (\beta_k^{(n)})^p \varepsilon_n$$

whence $\sum_k (\beta_k^{(n)})^p < \frac{1}{\varepsilon_n} \int \|x\|_{\ell_p}^p d\mu$. Now choose the numbers c_k , $k = 1, 2, \dots$, such that for each n there is a number λ_n satisfying the inequality $\lambda_n c_k > \beta_k^{(n)}$ and also $\sum_k c_k^p < \infty$. By construction, for each $\varepsilon > 0$ there is $\lambda_\varepsilon > 0$ such that $\mu\{x: |x_k| > \lambda_\varepsilon c_k\} \leq \varepsilon, k = 1, \dots$. Now consider the coincidence on linear functionals of the L_2 -topology and the topology of convergence in measure. This equivalence implies that for some $\varepsilon_0 > 0$ and any $k = 1, 2, \dots$, we have

$$\mu_{e_k}((- \varepsilon_0 \|e_k\|_{L_2(\mu)}, \varepsilon_0 \|e_k\|_{L_2(\mu)})) < 1 - \varepsilon_0.$$

If now the ellipsoid $\mathcal{E}$, defined above were not absorbed by the parallelepiped with vertex (c_k) , this would imply that, conversely, for any $\varepsilon > 0$ there is a number $k = k(\varepsilon)$ such that $c_k < \varepsilon \|e_k\|_{L_2}$. Now taking $\varepsilon = \frac{\varepsilon_0}{\lambda_{\varepsilon_0}}$ we obtain

$$\mu_{e_k}((- \varepsilon_0 \|e_k\|_{L_2(\mu)}, \varepsilon_0 \|e_k\|_{L_2(\mu)})) \geq \mu_{e_k}((- \lambda_{\varepsilon_0} c_k, \lambda_{\varepsilon_0} c_k)) > 1 - \varepsilon_0.$$

The contradiction establishes that the ellipsoid $\mathcal{E}$ is absorbed by the parallelepiped with vertex (c_k) , and since $(c_k) \in \ell_p$, this concludes the proof of the theorem.

If we now assume from the start that $\int \|x\|_{\ell_p}^p d\mu < \infty$, the general case is reduced to the previous case by introducing an appropriate density dependent on the norm which decreases at infinity. The Hilbert norms of linear functionals do not increase, and the convergence in measure is preserved. Q.E.D.

Remark. We have actually used the coincidence of the two topologies only on the sequence of the coordinate functionals e_k .

In conclusion, we give a simple example of a measure in ℓ_1 which is not concentrated on any oriented Hilbert subspace. Let Y be a random variable with probability distribution density $p(y) = y^{-2}$ for $y > 1$ and $p(y) = 0$ for $y \leq 1$. Using the three-series theorem, we easily check

that $\sum_k \frac{1}{k^{\frac{1}{4}}} Y_k < \infty$, a.c., where Y_k are independent identically distributed (with Y) random variables. Let the numbers $c_k > 0$ be such that $\sum_k c_k^2 < \infty$. We will show that the series $\sum \frac{Y_k^2}{c_k^2 k^{5/2}}$ does not converge with probability 1. Indeed, let $A_k = \{\omega: \frac{Y_k^2}{c_k^2 k^{5/2}} \leq 1\}$, then $PA_k = 1 - \frac{1}{c_k^2 k^{5/4}}$. We will show that the condition $\sum \frac{1}{c_k^2 k^{5/4}} < \infty$ (necessary for the convergence of the series $\sum \frac{Y_k^2}{c_k^2 k^{5/2}}$) by the three-series theorem is incompatible with the condition $\sum c_k^2 < \infty$. By the Holder inequality, we find

$$\sum_{k=1}^N \frac{1}{k^{5/6}} \leq \left[\sum_{k=1}^N \frac{1}{c_k^2 k^{5/4}} \right]^{\frac{2}{3}} \left[\sum_{k=1}^N c_k^2 \right]^{\frac{1}{3}}$$

whence it follows that the sums on the right cannot be bounded simultaneously for $N \rightarrow \infty$. The measure in ℓ_1 corresponding to the sequence $X_k = k^{-\frac{1}{4}} Y_k$ is the required measure. Note that the measure corresponding to independent random variables with second moments is always concentrated on some Hilbert subspace.

In conclusion, I would like to thank H. Sato and N. N. Vakhaniya for calling my attention to the error, and also V. N. Sudakov whose ideas (and in particular, his work [2]) provided the basis for the proof of the revised theorem and the construction of the example.

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FROM THE EDITORIAL BOARD

The Editorial Board requested Yu. V. Borovskikh to comment on the circumstances surrounding the publication of some of his articles in the series "Problems of the Theory of Probability Distributions." The following letter was received in reply:

"Two of my articles 'A New Method of Calculation of Exact Distributions in Nonparametric Statistics' and 'A Complete Asymptotic Expansion for Kolmogorov's Goodness of Fit Test' were published in Vol. 41 of *Zapiski Nauchnykh Seminarov LOMI* (English translation — J. Sov. Math., 9, No. 1 (1978)). In the first of the two articles, I failed to include a reference to an article by Yu. G. Reshetnyak (Dokl. Akad. Nauk SSSR, 130, No. 5, 981-983 (1960)) which had influenced my work.

In the second article, the list of references should include H. A. Lauwerier, Z. Wahr. Verw. Geb., 2, No. 1, 61-68 (1963).

In apologize for these omissions."

The English translation of *Zapiski* Vol. 41 published by Plenum Publishing Corporation includes the revised text.