Erratum

Kossakowski, A., Frigerio, A., Gorini, V., Verri, M.: Quantum Detailed Balance and KMS Condition. Commun. math. Phys. **57**, 97—110 (1977)

In the proof of Theorem 2.2, Equation (2.14) is wrong. To recover the correct proof, replace the part from line 1 after formula (2.13) ("Let $H_N = \dots$ ") to line -8 from the end of the proof ("...boundedness estimates") by the following:

Let

$$C_{rr'ss'} = \frac{1}{2}(K_{rr'ss'} + K_{s'sr'r}Q_s Q_s^{-1}) = K_{rr'ss'} + i(\varepsilon_r - \varepsilon_s)\delta_{rr'}\delta_{ss'},$$

the equality of the two expressions following from Equation (2.13). By construction, $\{C_{rr'ss'}\}$ satisfies (2.12), and moreover

$$\sum_{rr'ss'=1}^N C_{rr'ss'} P_{rr'} A P_{s's} = E_N \Psi E_N(A) + i[E_N(H), E_N(A)],$$

where $E_N(A) = Q_N A Q_N$, $Q_N = \sum_{r=1}^N P_{rr}$.

Notice that $E_N(A)$ converges ultraweakly to A as $N \rightarrow \infty$.

In particular, setting $A = \mathbb{1}$,

$$\sum_{rr'ss'=1}^N C_{rr'ss'} P_{rr'} P_{s's} = E_N \Psi(Q_N),$$

which converges ultraweakly as $N \rightarrow \infty$ to $\Psi(\mathbb{1})$, thus proving (2.11). Indeed,

$$\lim_{N \rightarrow \infty} \text{Tr} \{A_0 [E_N \Psi(Q_N) - \Psi(\mathbb{1})]\} = 0$$

for all A_0 in the linear span of the P_{rs} 's, which is dense in the space of trace class operators on $\mathcal{H}$, and $|\text{Tr} \{A_0 [E_N \Psi(Q_N) - \Psi(\mathbb{1})]\}|$ is bounded by $2 \|A_0\|_1 \|\Psi\|$.

Using the same kind of arguments, one proves that the expression

$$\begin{aligned} & \sum_{rr'ss'=1}^N C_{rr'ss'} (P_{rr'} A P_{s's} - \frac{1}{2} \{P_{rr'}, P_{s's}\} A) \\ &= E_N \Psi E_N(A) - \frac{1}{2} \{E_N \Psi(Q_N), A\} + i E_N([H, E_N(A)]) \end{aligned}$$

tends to $L_s(A)$ ultraweakly as $N \rightarrow \infty$.

Finally, for any finite sequence $\{x_{rr'}\}$ we have

$$\begin{aligned} & \sum_{rr'ss'=1}^N x_{rr'} C_{rr'ss'} x_{ss'} \\ &= \frac{1}{2} \sum_j \left\{ \left| \sum_{rr'} x_{rr'} \langle r' | V_j | r \rangle \right|^2 + \left| \sum_{rr'} y_{rr'} \overline{\langle r | V_j | r' \rangle} \right|^2 \right\} \geq 0, \end{aligned}$$

where $y_{rr'} = x_{rr'} (Q_r Q_r^{-1})^{1/2}$ (we have used the fact that $K_{rr'ss'} = 0$ if $Q_r Q_r^{-1} \neq Q_s Q_s^{-1}$). The inequality clearly holds also for those infinite sequences for which the expression converges. This proves (2.10).