Addendum to 'Asymptotic Eigenfunctions of Mixed Problems of Stekloff's Type': Extension to Exterior Problems

By Shawky E. Shamma, University of West Florida, Pensacola, Florida, USA

Let D be a simply connected bounded plane domain with regular boundary curve c and exterior domain $\tilde{D}$. We now consider the Stekloff problem for the domain $\tilde{D}$ exterior to the curve c ,

$$\Delta v = 0 \quad \text{in } \tilde{D} \cup \{\infty\} \quad (1)$$

$$\frac{\partial v}{\partial \nu} = \mu v(s)g(s) \quad \text{on } c, \quad (2)$$

where g is a sufficiently smooth, positive, periodic, and prescribed function of the arclength s , and ν is outward unit normal to D .

It has been shown by R.G. Kellner [1], that the special case $g(s) = 1$ of the above exterior Stekloff eigenvalue problem (1), (2) has an infinite sequence of eigenvalues tending to ∞ whose corresponding orthonormalized eigenfunctions form a complete set in $L_2(c)$. The above results can be generalized to the case where $g(s)$ is the above described function.

In this note we study the asymptotic behavior of the eigenfunctions on the curve c and the corresponding eigenvalues of the problem (1), (2).

Using the Green's identity

$$\iint_{\tilde{D}} [G\Delta v - v\Delta G]ds = \int_{\partial\tilde{D}} \left[G \frac{\partial v}{\partial \nu} - v \frac{\partial G}{\partial \nu} \right] ds$$

where $G(P, Q) = -1/2\pi \ln r(P, Q)$, is the free space Green's function, we get

$$v(P) = \int_c \left[G(P, q) \frac{\partial v(q)}{\partial \nu} - v(q) \frac{\partial G(P, q)}{\partial \nu} \right] ds_q, \quad P \in \tilde{D}.$$

Letting $P \rightarrow p$ on c , we find

$$\frac{v(p)}{2} = \int_c \left[G(p, q) \frac{\partial v(q)}{\partial \nu} - v(q) \frac{\partial G(p, q)}{\partial \nu} \right] ds_q. \quad (3)$$

Using (2) in (3), we get

$$\frac{v(s)}{2} = \int G(s, s') [\mu g(s') - K(s, s')] v(s') ds' \quad (4)$$

where $v(s) \equiv v(p)$, $G(s, s') \equiv G(p, q)$, and $K(s, s') = \partial G(p, q)/\partial \nu_q$.

The integral equation (4) is a special case, when $k(s) \equiv 0$, of an integral equation studied in [2]: (equation 1.5, page 2). Hence, we obtain the following result

$$\mu'_{2n} = \frac{2n\pi}{J} + \epsilon_n$$

$$v_{2n}(s) = \sqrt{\frac{2}{J}} \cos(2n\pi\xi + \kappa_n) + r_n(s)$$

and

$$\mu'_{2n+1} = \frac{2n\pi}{J} + \epsilon'_n$$

$$v_{2n+1}(s) = \sqrt{\frac{2}{J}} \sin(2n\pi\xi + \kappa_n) + r'_n(s),$$

$n = N, N+1, \dots$, where N is a positive integer chosen sufficiently large, the κ_n are constants independent of s , ϵ_n and ϵ'_n tend to zero as $n \rightarrow \infty$, $r_n(s)$ and $r'_n(s)$ tend to zero uniformly as $n \rightarrow \infty$, and

$$J = \oint_c g(s) ds, \quad \xi = \frac{1}{J} \int_0^s g(t) dt.$$

References

- [1] R. G. KELLNER, *Some Eigenvalue Problems Connected with Potential Theory*, Dept. of Math. Technical report, Stanford University, USA (1970).
- [2] S. E. SHAMMA, *Asymptotic, Eigenfunctions of Mixed Problems of Stekloff's Type*, Z. angew. Math. Phys. 23, 1-12 (1972).

Summary

The asymptotic eigenvalues and eigenfunctions of the problem: $\Delta V = 0$ in the exterior domain to a regular closed curve c and $\partial V / \partial \nu = \mu V(s)g(s)$ on c , are obtained. It is shown that $\mu_n = 0(n)$ and $V_n(s)$ becomes trigonometric.

Résumé

Soit D la région extérieure d'une courbe fermée régulière, c ; on détermine le comportement asymptotique des valeurs propres et fonctions propres du problème: $\Delta V = 0$ dans D et $\partial V / \partial \nu = \mu V(s)g(s)$ sur c . On peut démontrer que $\mu_n = 0(n)$ et les $V_n(s)$ deviennent des fonctions trigonométriques.

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