Errata to The slow motion of a sphere in a second-order fluid

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The following corrections should be introduced:

1. p. 165, statement after [2.13d] should read:
 “Especially for a spherical particle the last term of eq. [2.12] becomes $-6\pi\eta a^3 \dot{u}_{(0)h}^{(2)}$ ”.
2. p. 165: Replace the term $-8\pi\eta a^3 \dot{\omega}_h^{(2)}$ of [2.14] by $[\hat{\mathbf{F}} \cdot \dot{\mathbf{u}}_{(0)}^{(2)} + \hat{\mathbf{g}}(0) \cdot \dot{\boldsymbol{\omega}}]$.
3. p. 166, two lines before eq. [3.10]: “If $\kappa_0^{(i)} \mathbf{F}$ denotes the $\kappa_0^{(i)}$ contribution to the force which does not involve $\dot{\mathbf{u}}_{(0)}$ or $\dot{\boldsymbol{\omega}}$, then ...”.
4. p. 167, lines 10–15 on the left, should be replaced by: “Since $\int_{S_\infty} d^2x \mathbf{n} : (\mathbf{B} \cdot \mathbf{B}) = 0$ we can replace $\mathbf{f}^{(1)} \cdot \mathbf{f}^{(1)}$ in [3.20] by $\mathbf{f}^{(1)} \cdot \mathbf{f}^{(1)} - 36 \mathbf{B} \cdot \mathbf{B}$. Using Gauss’ theorem we can thus rewrite [3.20] as

$$\mathbf{e}_h \cdot \mathbf{F}^{(2,1)} = -2\eta \int_{V_f} d^3x (\mathbf{f}^{(1)} \cdot \mathbf{f}^{(1)} - 36 \mathbf{B} \cdot \mathbf{B}) : \frac{\partial \dot{\mathbf{u}}}{\partial \mathbf{r}}$$
”.
5. p. 167, eq. [3.24]: Replace the term $\mathbf{f}^{(1)} \cdot \mathbf{f}^{(1)}$ by $\mathbf{f}^{(1)} \cdot \mathbf{f}^{(1)} - 36 \mathbf{B} \cdot \mathbf{B}$.
6. p. 167, eq. [4.2]: Replace $\dot{\omega}_h^{(2)}$ by $\dot{\boldsymbol{\omega}}$.
7. In going from [3.11] to [3.12] the $\frac{\partial}{\partial t} \dot{p}^{(1)}$ from [3.11b] has been dropped. This, however, in general is not legal, for we are using a body-fixed coordinate system. Any undisturbed motion will thus become time dependent, even if this motion is steady with respect to an observer fixed in space. Consequently, the term to be added to [3.13] is

$$\int_{S_\infty} d^2x \eta_\mu \frac{\partial}{\partial t} \dot{\tau}_{\mu k}^{(1)}.$$

But since $\frac{\partial}{\partial x_\mu} \dot{\tau}_{\mu h}^{(1)} = 0$ we also have $\frac{\partial}{\partial x_\mu} \dot{\tau}_{\mu h}^{(1)} = 0$.

Consequently, the integration can be performed over any other surface completely enclosing the particle. Choosing S_0 instead of S_∞ and recalling that we are moving with the particle allows us to write this term as

$$\frac{\partial}{\partial t} \dot{\mathbf{F}}_h^{(1)},$$

where $\frac{\partial}{\partial t}$ is taken with respect to our co-rotational frame. This term should therefore be added to [3.14], so that eq. [4.1] becomes modified via addition of

$$\kappa_0^{(2)} \frac{\partial}{\partial t} \dot{\mathbf{F}}^{(1)}.$$

By the same token the correction to $\dot{\mathbf{g}}^{(2,2)}(0)$ appears as the additional term

$$\kappa_0^{(2)} \frac{\partial}{\partial t} \dot{\mathbf{g}}^{(1)}(0)$$

in [4.2].

Since external forces and/or torques are required for the sphere to move relative to the fluid the terms $\frac{\partial}{\partial t} \dot{\mathbf{F}}^{(1)}$ and $\frac{\partial}{\partial t} \dot{\mathbf{G}}^{(1)}(0)$ are non-zero.

If we have a homogeneous sphere and if the external influences ($\mathbf{f}^{(e)}$ and $\mathbf{g}^{(e)}$, respectively) are constant with respect to the laboratory frame then by means of the force and couple balance we get

$$\frac{\partial}{\partial t} \dot{\mathbf{F}}^{(1)} = - \frac{\partial}{\partial t} \mathbf{f}^{(e)} = \dot{\boldsymbol{\omega}} \times \mathbf{f}^{(e)},$$

$$\frac{\partial}{\partial t} \mathbf{g}^{(1)}(0) = - \frac{\partial}{\partial t} \mathbf{g}^{(e)} = \mathbf{\dot{\omega}} \times \mathbf{g}^{(e)}.$$

These terms (multiplied by $\kappa_0^{(2)}$) have to be included in the examples cited and the corresponding modifications are readily worked out. For example, if we have the sphere translating and rotating in a quiescent fluid only the classical expressions of *Stokes* emerge

(instead of [4.8]). The lift force talked about does not appear. It would appear, however, if the fluid instead of the sphere was rotating.

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