Corrections to "Leray-Schauder theory without local convexity"

By

VICTOR KLEE in Seattle

p. 287, line 12: Replace $[0, 1]$ by $[0, 1]$.

p. 288, line 6: Replace N by B .

p. 288, lines 20—29: Replace these lines ("Now it can be verified . . .") by the following:

Let μ denote the gauge functional of N and $K = \{(x, 0) : \mu x = 0\}$. It can be verified that each point $(x, t) \in (N \times [0, 1]) \sim K$ admits a unique expression in the form $(x, t) = r(x, t) y(x, t)$ for $r(x, t) \in [0, 1]$ and $y(x, t) \in Y$. Extending the functions $r(\cdot, \cdot)$ and $y(\cdot, \cdot)$ to K , we have the following formulae: when $x \in N$ and $t \in [0, 1]$, then

$$\begin{aligned} 0 < t &\leq \mu x & r(x, t) &= \begin{cases} t \\ x \in [0, 1] \\ 0 \end{cases} & \text{and} & y(x, t) = \begin{cases} (x/t, 1) \\ (x/\mu x, t/\mu x) \in Y \\ \text{arbitrary} \end{cases} \\ \text{if } 0 < \mu x &\leq t, & & & & \\ 0 = t = \mu x & & & & & \end{aligned}$$

For each $(x, t) \in N \times [0, 1]$, define

$$h^{**}(x, t) = (1 - r(x, t))q + r(x, t) h^*(y(x, t)).$$

Since N is shrinkable, μ is continuous, and consequently r is continuous on $N \times [0, 1]$ and y is continuous on $(N \times [0, 1]) \sim K$. Thus h^{**} is continuous on $(N \times [0, 1]) \sim K$. But as (x, t) approaches K , $r(x, t)$ approaches 0 and hence $r(x, t) h^*(y(x, t))$ approaches $0 = h^{**}K$, for the set h^*Y is bounded. Thus h^{**} is continuous on all of $N \times [0, 1]$. Clearly $h^{**} \supset h^*$ and it is easy to verify that h^{**} is compact; that is, $h^{**} \in C(N \times [0, 1], E)$.

p. 288, line 3*: Replace $q = 0$ by $\mu x = 0$.

p. 290, line 4: Replace ζ_0 by ζ .

p. 290, line 17*: The sentence "If $x \in \text{int} N_0 \dots$ " should be replaced by the following:

We claim that $\eta x \notin [0, 1] \eta(-x)$. To prove this, it suffices to consider the case in which $x \in N_0$, for if $x \notin N_0$ and $x \in [0, 1] \eta(-x)$, then $f_0(x/\mu x) \in [0, 1] f_0(-x/\mu x)$ and of course $x/\mu x \in N_0$.

p. 290, line 11*: Replace $\eta_t x = 0$ by $\eta_t x \neq 0$.

p. 291, line 5: Replace "thal" by "that".

p. 291, line 8: Replace $Z \bigcup_{x \in N} [\xi x, -\xi(-x)]$ by $Z = \bigcup_{x \in N} [\xi x, -\xi(-x)]$.

p. 291, lines 12—16: Replace these lines ("Then r is . . .") by the following:

*) VICTOR KLEE, Leray-Schauder theory without local convexity. Math. Ann. 141, 286—296 (1960).

Then r is a continuous retraction of kN onto N . With B replaced by kB and φ by ξr , the conditions of 1.5 are satisfied and therefore ξr admits a fixed point $y_0 \in kN \sim kB$. With $\xi r y_0 = y_0$, we have $r \xi r y_0 = r y_0 \in N$. Now we claim $\xi r y_0 \in N$. For if $\xi r y_0 \notin N$, then $r \xi r y_0 \in B$ by the definition of r , whence $r y_0 \in B$ and $\xi r y_0 \in N$ by hypothesis. Since $\xi r y_0 \in N$, we have $r \xi r y_0 = \xi r y_0$ and consequently the point $x_0 = r y_0$ is fixed under ξ .

p. 291, line 23: Replace $r\eta$ by ηr_1 .

(Received October 1st, 1961)