Erratum

On the K -theory of the classifying space of a discrete group

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A typographical error occurred in the statements and proofs of the main results (Theorem 3.1 and Theorem 4.2) on pages 320, 324, and 326. They should read as follows:

On page 320:

Theorem 3.1.

$$K_G^*(X/\Gamma') \otimes \mathbb{C} \cong K^*(B\Gamma')^G \otimes \mathbb{C} \oplus \bigoplus_{\gamma \in \mathcal{C} - \{1\}} K^*(B(C(\gamma) \cap \Gamma'))^{H_\gamma} \otimes \mathbb{C}$$

where $\mathcal{C}$ is the set of conjugacy classes of elements of finite order in Γ , and $H_\gamma = C(\gamma)/C(\gamma) \cap \Gamma'$, a finite group.

On page 324:

Theorem 3.1.

$$K_G^*(X/\Gamma') \otimes \mathbb{C} \cong \bigoplus_{(\gamma)} K^*(B(C(\gamma) \cap \Gamma'))^{H_\gamma} \otimes \mathbb{C}$$

where γ ranges over conjugacy classes of elements of finite order in Γ , and $H_\gamma = C(\gamma)/C(\gamma) \cap \Gamma'$, a finite group.

Proof. Simply combine (2.2) with (1.4) and identify

$$\begin{aligned} K^*(X^{(\gamma)}/C(\gamma)) \otimes \mathbb{C} &= K^*([X^{(\gamma)}/C(\gamma) \cap \Gamma']/H_\gamma) \otimes \mathbb{C} \\ &= K^*(B(C(\gamma) \cap \Gamma'))^{H_\gamma} \otimes \mathbb{C}. \end{aligned}$$

On page 326:

Theorem 4.2.

- (i)
$$K_p^*(B\Gamma) \otimes \mathbb{C}_p \cong \bigoplus_{\substack{(\gamma) \\ \gamma \in \Gamma(p)}} K_p^*(B\Gamma' \cap C(\gamma))^{H_\gamma} \otimes \mathbb{C}_p$$
- (ii)
$$\chi(K_p^*(B\Gamma) \otimes \mathbb{C}_p) = \sum_{\substack{(\gamma) \\ \gamma \in \Gamma(p)}} \tilde{\chi}(C(\gamma)).$$

Example 4.3. Now let $\Gamma = \mathrm{SL}_3(\mathbb{Z})$, $\Gamma' = \Gamma(3)$, $G = \mathrm{SL}_3(\mathbb{F}_3)$. Note $|G| = 2^4 \cdot 3^3 \cdot 13$ but that Γ has no 13-torsion. Hence we deduce

$$K_p^*(B\Gamma) \otimes \mathbb{C}_p \cong \bigoplus_{\substack{(\gamma) \\ \gamma \in \Gamma(p)}} K^*(BC(\gamma) \cap \Gamma(3))^{H_\gamma} \otimes \mathbb{C}_p \quad \text{for } p = 2, 3.$$

The statement at the end of the Example 4.3 is inaccurate. It should be modified to say ...“the centralizers of elements of order 2 or 4 in $\mathrm{SL}_3(\mathbb{Z})$ are rationally acyclic”... The conclusions remain valid.

Finally, it has been brought to the author’s attention that the result attributed to Atiyah and Segal on p. 320 of [A-S2] was proved by N.J. Kuhn in: Character rings in algebraic topology. Advances in Homotopy Theory (London Math. Soc. Lect. Note Ser., vol. 139), Salamon, Steer and Sutherland (eds.). Cambridge: Cambridge University Press 1989.