A correction

to the paper "On the Stability of Certain Finite Difference Approximations to Parabolic Systems of Differential Equations"

by D. G. ARONSON

Numer. Math. **5**, 118—137 (1963)

The proof of Lemma 2.1 in the above named paper is erroneous. However, the lemma itself is correct as we shall now prove. In particular, equation (2.8) is not valid unless the matrices $\tilde{A}_1$ and $\tilde{A}_2$ commute. Since there is no reason to suppose that these matrices are commutative, it is necessary to supply an alternative for that part of the proof which is based on (2.8), viz, the estimation of

$$\tilde{A}^p \equiv (\tilde{A}_1 + \lambda |\sin \alpha h|_2^{2b} \tilde{A}_2)^p$$

for $p > N - 1$.

Let $\kappa = \lambda |\sin \alpha h|_2^{2b}$. Then

$$(1) \quad \tilde{A}^p = \sum_{j=0}^p \kappa^j \sum_{l=1}^{\binom{p}{j}} T^{(jl)},$$

where each $T^{(jl)}$ is a product of $p-j$ $\tilde{A}_1$'s with j $\tilde{A}_2$'s. Recall that $\tilde{A}_1$ is a diagonal matrix with elements $\delta_{rs} \mu_r$ and that $\tilde{A}_2$ is a lower triangular matrix with zero main diagonal. The non-zero elements of $\tilde{A}_2$ are $\bar{a}_{rs}$, where $|\bar{a}_{rs}| \leq \varepsilon$. Write $T^{(jl)} = (T_{rs}^{(jl)})$ for $r, s = 1, 2, \dots, N$. In view of the form of the matrices $\tilde{A}_1$ and $\tilde{A}_2$ it follows that $T_{rs}^{(jl)} = 0$ for $r < s + j$. Thus $T^{(jl)} = 0$ for $j > N - 1$ and the outer sum in (1) actually goes from $j=0$ to $j=N-1$. Moreover, for $r \geq s + j$ and $j \leq N - 1$, $T_{rs}^{(jl)}$ is a $(j-1)$ -fold sum of products of j $\bar{a}_{rs}$'s with $p-j$ μ_r 's. Since we have shown that $|\mu_r| \leq \exp\{-\frac{1}{2}M(\lambda)\kappa\}$, we obtain

$$(2) \quad |T^{(jl)}| \leq \varepsilon^j N^j \exp\left\{-\frac{(p-j)}{2} M \kappa\right\}.$$

Thus

$$\begin{aligned} |\tilde{A}^p| &\leq \sum_{j=0}^{N-1} \binom{p}{j} (\kappa \varepsilon)^j N^j \exp\left\{-\frac{(p-j)}{2} M \kappa\right\} \\ &\leq N^{N-1} \exp\left\{-\frac{p-N+1}{2} M \kappa\right\} \sum_{j=0}^{N-1} \frac{(p \kappa \varepsilon)^j}{j!} \\ &\leq N^{N-1} \exp\left\{-p \kappa \left(\frac{M}{2} - \varepsilon\right) + \frac{M}{2} (N-1) \kappa\right\} \end{aligned}$$

and (2.11) follows as before.

Department of Mathematics
Institute of Technology
University of Minnesota
Minneapolis, Minnesota

(Received July 1, 1963)