Correction

to the Contribution SHREERAM ABHYANKAR in Lafayette:

Uniformization of Jungian Local Domains

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In the above cited paper we used the treatment of the second chain condition given in NAGATA [15]. We have recently noticed that in NAGATA [16: lines 1 to 4 on page 219] it is pointed out that the said treatment is not quite correct. Here we shall rectify this situation.

First we take the opportunity to correct some misprints: In line 30 of page 3 replace $\dim_R A$ by $\dim_A R$. In line 21 of page 21 replace 1) by (1). On page 25 the displayed formulas should be numbered as 1), 2), 3) instead of (1), (2), (3) respectively. On page 26 everywhere replace (1), . . . , (8) by 1), . . . , 8) respectively, except: in lines 4 and 29 do not change (1). On page 27 everywhere replace (1), . . . , (16) by 1), . . . , 16) respectively, except: in lines 11 and 32 do not change (1); in line 28 do not change (2); and in line 29 do not change (3). In the proof of Theorem 10 on pages 29 and 30 everywhere replace (1), . . . , (8) by 1), . . . , 8) respectively. In line 15 of page 31 replace $(x, y)N^*$ by $(x, y)S^*$. In line 22 of page 31 replace (x', Y) by $(x', Y)R^*$.

The said rectification is thus. The material which begins on line 32 of page 5 and ends on line 19 of page 6 and which now reads "The *dimension formula* . . . get the following" should be replaced by the following material:

The *dimension formula* is said to hold in a noetherian domain A if for every spot (R, M) over A we have that: $\dim R + \dim_A R = \dim A_{A \cap M} + \text{trdeg}_A R$. By a *maximal chain* of prime ideals in a domain A we mean a finite sequence $P_0 \subset P_1 \subset \cdots \subset P_d$ of distinct prime ideals in A such that: $P_0 = \{0\}$; P_d is a maximal ideal in A ; and there does not exist any prime ideal P in A such that $P_{i-1} \subset P \subset P_i$ and $P_{i-1} \neq P \neq P_i$ for some i with $1 \leq i \leq d$. The *first chain condition* is said to hold in a noetherian domain A if for every maximal chain of prime ideals $P_0 \subset P_1 \subset \cdots \subset P_d$ in A we have that $d = \dim A_{P_d}$. The *second chain condition* is said to hold in a noetherian domain A if for every non-negative integer n the first chain condition holds in $A[X_1, \dots, X_n]$ where $X_1, \dots, X_n$ are indeterminates (note that $A[X_1, \dots, X_n] = A$ in case $n = 0$). The significance of this condition is explained by the following proposition.

Proposition 4. *For any noetherian domain A the following three conditions are equivalent: (1) The second chain condition holds in A . (2) If P is any prime ideal in A , B' is any affine ring over A/P , H is any multiplicative set in B' , and B is the quotient ring of B' with respect to H , then the first chain condition holds in B . (3) The first chain condition holds in A , and for every prime ideal P in A the dimension formula holds in A/P .*

For a proof see 5.6 of Chapter IV of Grothendieck's *Éléments de Géométrie Algébrique*. A proof can also be deduced from Appendix 1 of ZARISKI-SAMUEL [24]. From Proposition 4 we get

Proposition 5. *If the second chain condition holds in a noetherian domain A then the second chain condition and the dimension formula hold in every spot over A .*

By NAGATA [16: (25.10), (25.12), (34.8)] we get

Proposition 6. *The second chain condition holds in every Dedekind domain.*

Let R be a two dimensional local domain such that the integral closure S of R in its quotient field is a local domain. Then by ZARISKI-SAMUEL [23: § 2 of Chapter IV] we get that $\dim S = 2$ and $\dim S_Q = \dim R_{R \cap Q}$ for every prime ideal Q in S . Let n be any nonnegative integer and let $A = R[X_1, \dots, X_n]$ and $B = S[X_1, \dots, X_n]$ where $X_1, \dots, X_n$ are indeterminates. Let $P_0 \subset P_1 \subset \dots \subset P_d$ be any maximal chain of prime ideals in A . Let $M = P_d$ and $P = M \cap R$. By ZARISKI-SAMUEL [23: § 2 of Chapter IV] there exists a maximal chain of prime ideals $Q_0 \subset Q_1 \subset \dots \subset Q_d$ in B such that $Q_i \cap A = P_i$ for $0 \leq i \leq d$. Let $N = Q_d$ and $Q = N \cap S$. Since N is a maximal ideal in B , by the Hilbert Nullstellensatz we get that $\dim_S B_N = 0$. Since S is a two dimensional normal local domain, by NAGATA [16: (25.10), (25.13), (34.8)] we get that the second chain condition holds in S and hence $d = \dim B_N$. Since the second chain condition holds in S , by Proposition 5 we get that $\dim B_N + \dim_S B_N = \dim S_Q + \text{trdeg}_S B$. Since $Q \cap R = P$ we conclude that $d = n + \dim R_P$. We can take a maximal chain of prime ideals $P'_0 \subset P'_1 \subset \dots \subset P'_e$ in A such that $P'_e = M$ and $e = \dim A_M$, and then by what we have just proved we get that $e = n + \dim R_P$. Therefore $d = \dim A_M$. Thus the second chain condition holds in R . Therefore by Proposition 5 we get the following.

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