Erratum

C. Peters: “The Local Torelli Theorem I. Complete Intersections.” Math. Ann. **217**, 1—16 (1975)

Sampei Usui noticed a mistake in the proof of Lemma (6.2). This lemma is used only in the proof of (6.4) for which I shall give another and even simpler proof. This reduces Macaulay-type of theorems to certain easily verified vanishing theorems. Recall the statement of Lemma (6.4):

Let $J_k: \bigoplus_{j=1}^{n+1} \sigma_k \rightarrow \bigoplus_{j=1}^s \sigma_n$ be given by the jacobian matrix $(\partial_i \phi_j)$, then for $k \leq \lambda + 1$ the kernel of J_k modulo $\mathfrak{p}$ consists (mod $\mathfrak{p}$) of $\{(F\xi_0, \dots, F\xi_n) | F \in \sigma_{k-1}\}$.

(1) This is equivalent to the vanishing of $H^0(T_V(k))$ for $k \leq \lambda + 1$.

(This remark is due to S. Usui.)

Proof. One may construct for each k a diagram D_k analogous to the ones shown for $k=0$, λ in Proposition (5.2). The only difference is that the map-say j_k -which plays the role of j and $j \otimes 1$ respectively is not injective in general, its kernel being $H^0(T_V(k))$. Since j_k is a factorisation of the map J_k , chasing in the diagram D_k shows the required equivalence.

(2) It suffices to show that $H^0(T_V(\lambda + 1)) = 0$.

Indeed, $H^0(T_V(k))$ is dual to $H^{1,m}(O_V(\lambda - k))$ and this space is zero for $k < \lambda$ (by “Nakano”) as well as for $k = \lambda$ [cf. Lemma 3.3 (iii)].

(3) Since $T_V(\lambda) \cong \Omega_V^{m-1}$, $m = \dim(V)$, one needs to see that $H^0(\Omega_V^{m-1}(1)) = 0$. To this end we define for any submanifold V of $\mathbb{P}_n$:

$$I(V): H^p(\Omega_V^q(k)) = 0 \quad \text{for } p + q \leq m - 1; \quad (p, q) \neq (0, 0) \quad \text{and } k = 1 \quad \text{or } k < 0 \\ (m = \dim V).$$

Notice that $I(\mathbb{P}_n)$ holds in view of “Bott’s theorem”, so we can start an inductive procedure thanks to:

Lemma. If $V = W \cap H$ is a smooth hypersurface section of a manifold W in $\mathbb{P}_n$ by a hypersurface H of degree $d \geq 2$ then $I(W)$ implies $I(V)$.

Proof. The exact sequences:

$$0 \rightarrow \Omega_V^{p-1}(-d+k) \rightarrow \Omega_W^p(k)|_V \rightarrow \Omega_V^p(k) \rightarrow 0$$

inductively show that

$$H^i(\Omega_V^p(k))=0, \quad \text{if} \quad H^{i+i}(\Omega_W^{p-i}(k-id)|V)=0 \quad \text{for} \quad i=0,\dots,p.$$

This in turn is true if

$$H^{i+i}(\Omega_W^{p-i}(k-id))=0 \quad \text{and} \quad H^{i+i+1}(\Omega_W^{p-i}(k-(i+1)d))=0 \quad \text{for} \quad i=0,\dots,p,$$

as can be seen by means of the exact sequences:

$$0 \rightarrow \Omega_W^{p-i}(k-(i+1)d) \rightarrow \Omega_W^{p-i}(k-id) \rightarrow \Omega_W^{p-i}(k-id)|V \rightarrow 0.$$

These remarks easily show that $I(V)$ is implied by $I(W)$.

Corollary. *If $V \subset \mathbb{P}_n$ is a complete intersection, then $H^0(\Omega_V^{m-1}(1))=0$, where $m = \dim V$.*