On the Maximum Modulus of Entire Functions and Their Derivatives: corrigendum

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(Received July 30, 1968)

Introduction: In an earlier paper ([2], Theorem(iii)) we have shown amongst various results the following:

$$M_{(s)}(r) < \mu(r)r^{(s+1)q-s+\varepsilon}, s = 1, 2, \dots, \quad (1)$$

for all $r > r_0$ (for notations and definitions see the above mentioned paper).

D. V. V. WEND [Math. Reviews, Vol. 33, 1454, (1967)] has noted an error in the proof of this result, although the result is correct — an error in the summation of an infinite series occurring in the proof. Although this error can be rectified, yet we wish to give a completely different proof of (1) depending on a well-known result of VALIRON.

Proof of (1): Let $\mu_{(s)}(r)$ and $\nu_s(r)$ correspond to the s -th derivative of $f(z)$ in the usual way (we follow the definition of $\mu_{(s)}(r)$ given in [1]). Then following VALIRON ([4], p. 32), one has:

$$M_{(s)}(r) < \mu_{(s)}(r) [2 \nu_s(r + r/\nu_s(r)) + 1].$$

However, to a given $\varepsilon > 0$ (ε is not necessarily the same at every step)

$$\nu_s(r) < r^{q+\varepsilon} \text{ for all } r \geq r_0 = r_0(\varepsilon). \quad (2)$$

Therefore, denoting by K various constants (> 0),

$$M_{(s)}(r) < K \mu_{(s)}(r) r^{q+\varepsilon}, \text{ for all } r \geq r_0(\varepsilon);$$

also ([3], Theorem 2)

$$\mu_{(s)}(r) \leq K \mu(r) \{\nu_s(r)/r\}^s,$$

and so using (2) again,

$$M_{(s)}(r) < K \mu(r) r^{e^s + e^{-s} + \varepsilon}, \text{ for all } r \geq r_0$$

and this leads to (1).

References

- [1] KAMTHAN, P. K.: On the maximum term of an integral function and its derivatives (II). *Mh. Math.* **68**, 125—129 (1964).
- [2] KAMTHAN, P. K.: On the maximum modulus of entire functions and their derivatives. *Mh. Math.* **69**, 407—409 (1965).
- [3] KAMTHAN, P. K. and P. C. DASH: On the derivatives of an entire function. *Bull. Acad. Polonaise Sci.*, **16**, 387—392 (1968).
- [4] VALIRON, G.: *Integral Functions*. New York: Chelsea Publications, 1949.

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