Correction to: “On Operators Derived from Extensions of the Fourier Transform”

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1. Introduction

In this note we make a number of comments which correct various mistakes in the paper [1] concerning the operator defined by

$$F_{\sigma}^{(v)}(f)(x) = (2\pi)^{-\frac{1}{2}} |x|^{v+\sigma} \int_{-\infty}^{\infty} |t|^v e^{ixt} f(t) dt,$$

whenever the integral exists for almost all x .

Our first proof of the boundedness of $F_{\sigma}^{(v)}$ from L^p into L^q , $0 < 1/p \leq 1$, $1/q = 1 - (1/p) - \sigma > 0$, $q \geq p$, $q > 1$, was incomplete. Hence the validity of the results for all v in the range $(1/p) - 1 < v < (1/p)$ is incorrect except when $F_{\sigma}^{(v)}$ is restricted to certain subsets of L^p .

A full discussion of these conclusions is given in [2].

We state below the results which we have so far proved concerning cases in which $F_{\sigma}^{(v)}$ is bounded on L^p .

2. The Main Theorems Involving Boundedness of $F_{\sigma}^{(v)}$

(2.1) (Theorem 6.1 and Note 6.1.1 of [2].) Let

$$1 < p \leq 2, \quad 1/q = 1 - (1/p) - \sigma > 0, \quad q \geq 2, \\ (1/p) - 1 < v \leq \min(0, -\sigma).$$

Then $F_{\sigma}^{(v)}$ can be extended to a bounded operator on L^p , there is a finite constant $k(p, v, \sigma) \equiv k$ such that

$$\|F_{\sigma}^{(v)}(f)\|_q \leq k \|f\|_p, \quad (f \in L^p),$$

and

$$(2.1.1) \quad \lim_{a, b \rightarrow \infty} \int_{-a}^b F_{\sigma}^{(v)}(f)(x) - (2\pi)^{-\frac{1}{2}} |x|^{v+\sigma} \int_{-a}^b |t|^v e^{ixt} f(t) dt \Big| dx = 0.$$

Also for g in $\mathcal{L}'$, we have

$$\|F_{-\sigma}^{(v+\sigma)}(g)\|_{p'} \leq k \|g\|_{q'}.$$

Further, for f in L^p and g in $L^{q'}$,

$$(2.1.2) \quad \int_{-\infty}^{\infty} g(x) F_{\sigma}^{(v)}(f)(x) dx = \int_{-\infty}^{\infty} f(t) F_{-\sigma}^{(v+\sigma)}(g)(t) dt,$$

$$(2.1.3) \quad F_{\sigma}^{(v)}(f)(x) = (2\pi)^{-\frac{1}{2}} |x|^{v+\sigma} (d/dx) \int_{-\infty}^{\infty} |t|^v (e^{ixt} - 1) (it)^{-1} f(t) dt,$$

$$(2.1.4) \quad f(x) = (2\pi)^{-\frac{1}{2}} |x|^{-v} (d/dx) \int_{-\infty}^{\infty} |t|^{-v-\sigma} (e^{-ixt} - 1) (-it)^{-1} F_{\sigma}^{(v)}(f)(t) dt,$$

for almost all x .

(2.2) (Theorem 7.6 of [2].) Suppose that either of the following sets of conditions hold:

- (i) $1 < p < \infty$, $1/q = 1 - (1/p) - \sigma$, $\max(2, p) \leq q < 2p$, $(1/p) - 1 < v \leq (2/q) - 1$,
- (ii) $1 < q < \infty$, $1/q = 1 - (1/p) - \sigma$, $\max(\frac{1}{2}, 1/q) \leq 1/p < \frac{1}{2}(1 + 1/q)$, $(1/p) - 1 < v \leq (2/q) - 1 + \sigma$

then the same conclusions as in (2.1) hold.

(2.2.1) Note. The conditions (ii) of (2.2) are not given explicitly in [2]. They may easily be obtained by writing

$$p \text{ for } q', \quad q \text{ for } p', \quad -\sigma \text{ for } \sigma, \quad \text{and} \quad v \text{ for } v + \sigma$$

in the conjugate inequality involving $F_{-\sigma}^{(v+\sigma)}$.

(2.3) (Theorems 8.1 and 8.2 of [2].)

(i) Let $1 < p < \infty$, $\tau = 1 - (2/p)$, $(1/p) - 1 < v \leq \min(0, (2/p) - 1)$. Then the conclusions of (2.1) hold with $\sigma = \tau$, $q = p$.

(ii) Let $f \in L^p$, $1 < p < \infty$, $\tau = 1 - (2/p)$, $\max(0, (2/p) - 1) \leq v < 1/p$, and suppose that $F_{\tau}^{(v)}(f) \in L^p$. Then there is a constant $k(p, v) \equiv k$ such that

$$\|f\|_p \leq k \|F_{\tau}^{(v)}(f)\|_p.$$

(2.4) Remark. We have shown in [2] by considering the example

$$f(t) = t^{-v} (1-t)^{-v}, \quad (0 < t < 1); \quad f(t) = 0, \quad t \notin (0, 1),$$

that $F_{\tau}^{(v)}$ is not bounded in L^p for $1/(2p) < v < (1/p)$. Further details of results so far proved can be seen in [2] and [4].

3. The Other Results of [1]

The conclusions given in Section 5 and 6 remain valid as they are, although the proofs given there need modification since $F_{\tau}^{(v)}$ is not bounded in L^p for all v in $((1/p) - 1, (1/p))$. Alternative proofs of those in Section 5 are contained in [3], and the conclusions of Section 6 may be justified similarly.

See remarks (9) of [2].

References

1. Okikiolu, G. O.: On operators derived from extensions of the Fourier transform. *Math. Z.* **92**, 175–186 (1966).
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3. — On certain extensions of the Hilbert operator. *Math. Ann.* **169**, 315–327 (1967).
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