Addendum

Representations of quantum algebras

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It has been pointed out to us (see the Introduction of [1]) that in Section 5 of the above paper we are implicitly using a non-obvious Mackey type result when we apply the results of Section 4. In [1], Lin proves in a more general context that one does indeed have such Mackey properties, at least over a base field (see Remark 2 below).

Here we show that the result we need follows in fact quite easily from arguments in our paper. Let the algebras U^b and $U^b(i)$, U_i , for $i = 1, \dots, n$, be defined as in our paper, see p. 4 and 2.5. (We take this opportunity to point out that, on p. 4, $U(I)$ should read $U^b(I)$.) Let U_i^b denote the subalgebra of U_i generated by $K_i^{\pm 1}$ and $F_i^{(r)}$, for $r \geq 0$. Then, with notation as in our paper, one has:

Proposition 5.0. *Let $M \in \mathcal{C}^b$. For any $i = 1, \dots, n$ and $r \geq 0$, one has natural isomorphisms of U_i -modules*

$$H^r(U^b(i)/U^b, M)|_{U_i} \simeq H^r(U_i/U_i^b, M|_{U_i^b}).$$

Proof. For $r = 0$ the statement follows directly from the definition of the induction functors in Section 2. To obtain it for all r it is enough (by the usual arguments based on the standard resolutions of 2.17) to check that $H^r(U_i/U_i^b, I(\mu)) = 0$, for $r > 0$, where $I(\mu)$ denotes $H^0(U^b/U^0, \mathcal{A}_\mu)$, for $\mu \in X$. It follows from the definition that $H^0(U_i/U_i^b, -)$ commutes with direct limits. Moreover, by 2.13 the category $\mathcal{C}_i^b$ has enough injectives and, using the fact that any object in $\mathcal{C}_i^b$ is a union of $\mathcal{A}$ -finite U_i^b -submodules, one obtains that a direct limit of injective objects in $\mathcal{C}_i^b$ is injective. By standard arguments it then follows that $H^r(U_i/U_i^b, -)$ commutes with direct limits, for $r \geq 0$. But $I(\mu)$ is the direct limit of U^b -submodules $V_m \simeq H^0(m\rho) \otimes \mathcal{A}_{m\rho+\mu}$, for $m \geq 0$, by Lemma 5.3 (note that this lemma does not use the results of 5.1–5.2) and hence, for $r \geq 0$, $H^r(U_i/U_i^b, I(\mu))$ is the direct limit of $H^r(U_i/U_i^b, V_m)$, for $m \rightarrow \infty$. Finally, using Corollary 3.3 (i) and Proposition 2.16, one obtains

$H^r(U_i/U_i^b, V_m) \simeq H^0(m\rho) \otimes H^r(U_i/U_i^b, \mathcal{A}_{m\rho+\mu})$ and, by Proposition 4.2, this vanishes for $r > 0$ and $m \gg 0$. The proposition follows.

Remarks. 1. The proposition is used implicitly in 5.1, 5.4–5.8.

2. Let $I \subseteq \{1, \dots, n\}$. Using Corollary 5.7, applied to the algebra U_I , one then obtains that the proposition generalizes to the case of induction from U^b to $U^b(I)$. Over a base field this is proved by different arguments in [1].

[1] Lin, Z.: A Mackey Decomposition Theorem and Cohomology for Quantum Groups at Roots of 1. *J. Algebra* **166**, 100–129 (1994)