Erratum: Symmetric Iterative Interpolation Processes

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As Professor Charles A. Micchelli has kindly pointed out to us, the proof of Theorem 6.4 in [1] is incorrect. We state again this theorem and we supply a satisfactory proof. We use the same notation, all references in the proof use previous results of the same article.

Theorem 6.4. *The fundamental function $F(t)$ of an interpolation process of type (b, N) has a unique continuous extension to the real axis. The Fourier transform of that extension is $G(\xi)$.*

Proof. Since $G(\xi)$ is integrable (see Theorem 6.3), we define the continuous function $H(x) = (1/2\pi) \int G(\xi) e^{ix\xi} d\xi$. The basic step of the proof is to show that $H(m) = F(m)$ for any integer m . Let us consider the following sequence of functions: $f_0(\xi) \equiv 1$ and, for $n > 0$, $f_n(\xi) = \prod_{k=1}^n [P(-\xi/b^k)/b]$ if $|\xi| \leq \pi b^n$, otherwise $f_n(\xi) = 0$. From the fact that $G(\xi) = \prod_{k=1}^{\infty} [P(-\xi/b^k)/b]$, an absolutely convergent infinite product, the zeros of G are those of its factors $[P(-\xi/b^k)/b]$; it follows that $G(\xi)$ is positive on $[-\pi, \pi]$. If the number A is chosen as

$$\sup\{f_n(\xi): \xi \in [-\pi, \pi], n = 0, 1, 2, \dots\} / \min\{G(\xi): \xi \in [-\pi, \pi]\},$$

then $f_n(\xi) \leq AG(\xi)$, for $n = 0, 1, 2, \dots$, and for any ξ in $[-\pi, \pi]$. If ξ is a number in $[-\pi b^m, \pi b^m]$ and if $n \geq m$, then

$$f_n(\xi) = \prod_{k=1}^m [P(-\xi/b^k)/b] f_{n-m}(\xi/b^m) \leq \prod_{k=1}^m [P(-\xi/b^k)/b] AG(\xi/b^m) = AG(\xi).$$

The last equality is a consequence of the functional equation (5.2). The sequence $\{f_n\}$ is dominated by the integrable function AG . From the Lebesgue dominated convergence theorem, $H(m)$ is the limit of $(1/2\pi) \int_{-\pi b^n}^{\pi b^n} f_n(\xi) e^{im\xi} d\xi$ when n tends to

∞ . According to formula (5.1), $\prod_{k=0}^{n-1} P(b^k \theta) = \sum_k F(k/b^n) e^{ik\theta}$, so

$$\frac{1}{2\pi} \int_{-\pi b^n}^{\pi b^n} f_n(\xi) e^{im\xi} d\xi = F(m) \quad \text{and} \quad H(m) = F(m).$$

From the functional relation (5.2), $G(b\xi) = G(\xi)P(-\xi)/b$, it follows that $H(x/b) = \sum_k F(k/b)H(x-k)$. By induction, $H(x) = F(x)$ for any b -adic number. H is the continuous extension of F and H has compact support, it is integrable, and its Fourier transform cannot be different from G . ■

Reference

1. G. DESLAURIERS, S. DUBUC (1989): *Symmetric iterative interpolation processes*. Constr. Approx. 5:49–68.