ERRATA

The following is a replacement for page 20 of the article entitled *Minimax Controller Design Using Rate Feedback* by N.K. Bose and C. Charoenlarnopparut which appeared in *Circuits, Systems, and Signal Processing*, Volume 18, Number 1, pp. 17–25, 1999.

of the Lagrange multipliers. Taking the partial derivatives of G with respect to $A(\cdot)$, $B(\cdot)$, $C(\cdot)$, $D(\cdot)$, and $E(\cdot)$, the resulting set of linear equations is written in matrix-vector form as

$$\begin{bmatrix} 2(A+C) & 2(AD+BC) & 0 & 2A \\ 2 & 2(AC+B+D) & 2BD & 0 \\ 2A & 2AB & 0 & -2C \\ 1 & A^2+2B & B^2 & 0 \\ 0 & 0 & 0 & 2E \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (8)$$

Denote the coefficient matrix of (8) by $\mathbf{X}$, and define

$$\Lambda \triangleq [\lambda_1 \ \lambda_2 \ \lambda_3 \ \lambda_4]^t, \quad \mathbf{y} \triangleq [1 \ 0 \ 0 \ 0 \ 0]^t,$$

so that (8) may be expressed in the compact form

$$\mathbf{X}\Lambda = \mathbf{y}.$$

For consistency, the rank of the coefficient matrix $\mathbf{X}$ must equal the rank of the augmented square matrix $\mathbf{Y} \triangleq [\mathbf{X} \ \mathbf{y}]$. From the structure of $\mathbf{Y}$ it is then clear that a nontrivial solution vector $\Lambda = [\lambda_1 \ \lambda_2 \ \lambda_3 \ \lambda_4]^t$ will exist if and only if the matrix $\mathbf{Y}$ is singular. This implies that the condition

$$8A^2BE(AD - BC) = 0$$

must hold. Because $A \neq 0$, $B \neq 0$, therefore either $AD - BC = 0$ or $E = 0$. We are now ready to supply the recursive procedure for construction of the minimax solution.

Case 1. The constraint $AD - BC = 0$, along with equations (4), (5), (6), and (7), lead to a solution set $\{A, B, C, D, E\}$, which can be calculated in a recursive manner as described next. The parameters B, A, D, C , and E are computed successively as real positive-valued solutions, if those exist (otherwise, recourse to Case 2 below is required), from the equations that follow. These equations are generated by constructing the Gröbner basis for the ideal generated by polynomials in equations (4), (5), (6) and $AD - BC = 0$, with respect to the lexicographical order [4], implemented by the symbolic algebra program Mathematica using the following command line:

GroebnerBasis[{D+2AC+A^2+2B-c1, A^2D+2BD+2ABC+B^2-c2, B^2D-c3, AD-BC}, {B, C, D, A}].

$$B^6 - c_2B^4 - 2c_3B^3 + 3c_1c_3B^2 - 2c_2c_3B + c_3^2 = 0 \quad (9)$$

$$A = \sqrt{\frac{B(-B^3 + c_2B - 2c_3)}{3c_3}} \quad (10)$$

$$D = \frac{c_3}{B^2} \quad (11)$$

$$C = \frac{AD}{B} \quad (12)$$

$$E = \sqrt{C^2 - A^2}. \quad (13)$$