ERRATA

D. XIA, Analytic Theory of Subnormal Operators,
IEOT (1987), 880-903

In the paper [2], the line 9 of p. 900 in general is not correct. However, this part of proof (line 8 to line 13 of p. 900 [2]) can be easily changed by the following.

Since the function $\xi_j(\zeta)$, $\zeta \in \mathbf{T}$ is the boundary value of a matrix-valued meromorphic function on $D = \{\zeta : |\zeta| \leq 1\}$ with a finite set of poles in $\{\zeta : |\zeta| < 1\}$, there is a rational function $r_j(\cdot)$ satisfying

$$r_j(\zeta) > 0, \quad |\zeta| = 1$$

such that the function $\eta_j(\zeta) = \xi_j(\zeta)r_j(\zeta)$ is regular on $\{\zeta : 0 < |\zeta| \leq 1\}$ with possible pole at $\zeta = 0$. Since the matrix-valued function $\eta_j(\zeta) \geq 0$ on $|\zeta| = 1$, by a generalized Herglotz theorem, there is a matrix-valued polynomial $p(\cdot)$ such that

$$\eta_j(\zeta) = p\left(\frac{1}{\zeta}\right)^* p(\zeta). \quad (1)$$

The existence of the factorization (1) may be proved by applying Wiener and Akutowicz theorem [1] to $\eta(\zeta) + \epsilon I$, $\epsilon > 0$ and then letting $\epsilon \rightarrow 0$. The rank of $\eta_j(\cdot)$ equals $m_j \leq n$ for a.e. ζ . Therefore $\text{rank } p(\zeta) = m_j$ for a.e. ζ .

Let $p(\zeta) = (p_{ik}(\zeta))$. We may assume that $p_j(\zeta) = (p_{ik}(\zeta))_{\substack{i=1,2,\dots,m_j \\ k=1,2,\dots,n}}$ is of rank m_j and there is a rational function $\alpha(\zeta) = (\alpha_{ik}(\zeta))_{\substack{i=1,\dots,n \\ k=1,2,\dots,m_j}}$ satisfying $\alpha_{ik}(\zeta) = 0$ for $i \neq k$ and $i \leq m_j$ and $\alpha_{ii}(\zeta) = 1$ for $i = 1, 2, \dots, m_j$ such that

$$p(\zeta) = \alpha(\zeta)p_j(\zeta).$$

It is easy to see that $\alpha^* \alpha \geq I_{m_j}$ where I_{m_j} is the $m_j \times m_j$ identity matrix. By Wiener and Akutowicz theorem again, there is a $m_j \times m_j$ matrix-valued analytic function $\beta_j(\cdot)$ such that

$$\alpha(\zeta)^* \alpha(\zeta) = \beta_j(\zeta)^* \beta_j(\zeta), \quad |\zeta| = 1.$$

Therefore $\eta_j(\zeta) = q_j(\frac{1}{\zeta})^* q_j(\zeta)$, where $q_j(\zeta) = \beta_j(\zeta)p_j(\zeta)$ is a $m_j \times n$ matrix-valued analytic function. Let M_j be an m_j dimensional subspace of M . Using Blaschke product, it is easy to see that there is an $\mathcal{L}(M \rightarrow M_j)$ valued analytic function $f_j(\zeta)$ on $|\zeta| < 1$ such that

$$\xi_j(e^{i\theta}) = f_j(e^{i\theta})^* f_j(e^{i\theta})$$

which is the replacement of the factorization in line 13 of p. 900 [2].

References

- [1] Wiener, N. and Akutowicz, E. J., Factorization of positive Hermitian matrices, Wiener, N., Collected Works III, (1981) 264-275. MIT Press.
- [2] Xia, D., Analytic theory of subnormal operators, Integral Equations and Operator Theory, 10(1987) 881-903.