Correction to:

Homological Dimension and Stationary Sets

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Michael Höppner has kindly pointed out to me that the proof of Lemma 1.2 of my paper is not correct; the main problem is that $f(A) \cap B_{v_n}$ need not be κ -generated. A correct version of the lemma reads as follows.

1.2. Lemma. *Let κ be a regular uncountable cardinal. Let $A = \bigcup_{v < \kappa} A_v$, $B = \bigcup_{v < \kappa} B_v$ be κ -filtrations of κ^+ -generated modules A and B and let $f: A \rightarrow B$ be a homomorphism. Then there is a normal function $\phi: \kappa \rightarrow \kappa$ such that for all $v < \kappa$, $f(A_{\phi(v)}) \subseteq f(A) \cap B_{\phi(v)}$. If, moreover,*

(*) *for all $v < \kappa$, $f(A) \cap B_v$ is κ -generated,*

then we can choose ϕ so that for all $v < \kappa$, $f(A_{\phi(v)}) = f(A) \cap B_{\phi(v)}$.

Of the three applications of Lemma 1.2, only the second one (in the middle of p. 5) is affected. In order to fix this, the hypotheses of Theorem 1.4 (hence also of 1.6 and 1.7) must be strengthened to: “ R is κ -coherent and M is a κ^+ -generated module with a κ -filtration $M = \bigcup_{v < \kappa} \tilde{M}_v$ such that each $\tilde{M}_v$ is κ -presented.”

Notice that by Lemma 1.2 (with $f = \text{identity}$) any two filtrations of M agree on a cub, so the κ -filtration constructed in 1.4 can be taken to be $\{\tilde{M}_v: v \in \mathcal{C}\}$ for some cub $\mathcal{C}$. Now if $n > 0$ and 1.4 has been proved for $n-1$, choose for each v a κ -presentation

$$0 \longrightarrow K_v \longrightarrow F_v \xrightarrow{\pi_v} \tilde{M}_v \longrightarrow 0.$$

Let $F = \bigoplus_{v < \kappa} F_v$ and $\pi = \bigoplus_{v < \kappa} \pi_v: F \rightarrow M$. Then if $K = \text{Ker}(\pi) \subseteq F$, for all $v < \kappa$,

(*) $\tilde{K}_v \stackrel{\text{def}}{=} K \cap \bigoplus_{\mu < v} F_\mu$ is κ -generated.

Hence, $\tilde{K}_v$ is κ -presented (since R is κ -coherent). Thus the inductive hypotheses apply to K , so there is a cub $\mathcal{C}$ such that for all $v < \mu$ in $\mathcal{C}$, $\text{pd}(\tilde{K}_v) \leq n-1$ and

$\text{pd}(\tilde{K}_\mu/\tilde{K}_\nu) \leq n-1$. We can now apply Lemma 1.2 with $A=K$, $A_\nu=\tilde{K}_\nu$, $B=F$, $B_\nu=\bigoplus_{\mu<\nu} F_\mu$, $f=\text{inclusion}$, and finish the proof as before.

The applications of the Main Theorem are not affected; note that version III of the Main Theorem, on page 1, remains true as stated.

An additional correction: in Definition 1.3, " F is free of rank κ " should read " F is free of rank $<\kappa$ ".

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