Correction to “Collineations of Finite Projective Spaces as Permutations on the Sets of Dual Subspaces”

Math. Z. **111**, 249–254 (1969)

ASCHER WAGNER

I am indebted to H. Lüneburg for pointing out to me that expression (15) of the above paper is nonsense. This invalidates the next one and a half pages of that paper which are devoted to showing – which is crucial to the proof – that $\det(\mathbf{M}_t) \neq 0$.

In fact $\det(\mathbf{M}_t) \neq 0$ as we shall now show by proving that $\det(\mathbf{M}_t) = q^{(t+1)r}$. We first note that

$$q^{i(v-u+1)} b_{u,v+1} - q^i \cdot q^{i(v-u)} b_{u,v} = q^{(i+1)(v-u+1)} b_{u-1,v}$$

as may easily be verified by direct computation.

We now reduce $\det(\mathbf{M}_t)$ to triangular form as follows:

1th step. Subtract the k^{th} column from the $(k+1)^{\text{th}}$ column where k takes the value $t+1, t, \dots, 1$ in turn. (This replaces the first row by $1, 0, \dots, 0$, also it leaves the 1's on the main diagonal unchanged and replaces a typical term $b_{u,v}$ by $q^{v-u} b_{u-1,v-1}$.)

i^{th} step. Subtract q^{i-1} times the k^{th} column from the $(k+1)^{\text{th}}$ column where k takes the values $t+1, t, \dots, i$ in turn. (This replaces the i^{th} row by $0, \dots, 0, 1, 0, \dots, 0$ and replaces a typical term $q^{(i-1)(v-u)} b_{u-i+1,v-i+1}$ by $q^{i(v-u)} b_{u-i,v-i}$.)

After $t+1$ steps we obtain a triangular determinant with zeros above the main diagonal and diagonal entries $1, 1, \dots, 1, q^{(t+1)r} b_{-1,r-1}$.

Hence $\det(\mathbf{M}_t) = q^{(t+1)r} b_{-1,r-1} = q^{(t+1)r}$.

Dr. Ascher Wagner
 Queen Elizabeth College
 Campden Hill Road
 London W 8
 England

(Received March 31, 1971)