Corrections to
On Some Algebras of Analytic Functions
on Finite Riemann Surfaces

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Page 371, line 10 from the bottom of the page. The definition of F_j should read $F_j(\zeta_1, \dots, \zeta_N) = h_j(\zeta_1)$.

Page 372, line 10 from the top of the page. The statement "... if F is in $H_p[U^N]$, then $F \circ H$ is in $H_p[R]$ " is erroneous as has been pointed out to me by Prof. W. RUDIN who has communicated the following fact. Let $H[U]$ be the space of functions holomorphic in U , the open unit disc, and let

$$\mathfrak{B} = \{g \in H[U] : \iint_U |g(x + iy)|^2 dx dy < \infty\}.$$

Theorem. If for $f \in H_2[U^2]$ we define $f_D \in H[U]$ by $f_D(z) = f(z, z)$, then the map $f \rightarrow f_D$ carries $H_2[U^2]$ onto $\mathfrak{B}$.

Remark. Since $\mathfrak{B}$ is much larger than $H_2[U]$, this result shows that the quoted statement is in error.

Proof. If

$$f(z, w) = \sum_{m, n \geq 0} c_{mn} z^m w^n \quad \text{and} \quad f_D(z) = \sum_{k \geq 0} \gamma_k z^k,$$

then

$$\gamma_k = \sum_{m+n=k} c_{mn}$$

so

$$|\gamma_k|^2 \leq (k+1) \sum_{m+n=k} |c_{mn}|^2.$$

Thus,

$$\frac{1}{\pi} \iint_U |f_D|^2 = \sum_{k \geq 0} \frac{|\gamma_k|^2}{k+1} \leq \sum_{m, n \geq 0} |c_{mn}|^2 = \|f\|_2^2,$$

and consequently if $f \in H_2[U^2]$, $f_D \in \mathfrak{B}$.

On the other hand, if $g \in \mathfrak{B}$,

$$g(z) = \sum_{k \geq 0} \gamma_k z^k,$$

put

$$f(z, w) = \sum_{m, n \geq 0} \frac{\gamma_{m+n}}{1+m+n} z^m w^n.$$

Then $f \in H_2[U^2]$ and $g = f_D$. Thus, the map $f \rightarrow f_D$ is onto, and we have the result.

Apparently open is the problem of characterizing the functions f_D for $f \in H_p[U^2]$, $p \neq 2, \infty$.

In spite of the incorrect assertion contained in its proof, Theorem II.3 of the paper is correct as stated, and the indicated proof is sufficient. The proof suggested requires only a very special- and correct-case of the assertion, *viz*, that in which F is of the form $F(\zeta_1, \dots, \zeta_N) = h(\zeta_1)$ for $h \in H_p[U]$. For such a choice of F , $F \circ H$ is indeed in $H_p[R]$: If u is a harmonic majorant of F , then $u \circ \eta_1$ is a harmonic majorant for $F \circ H$ on R . The proof is now concluded as before.

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