Correction to "On the Radical of a Pseudo-Ring"

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In the above paper it is stated that condition (ii) of Patterson [1], Theorem 6, is superfluous. This is not the case; however, condition (ii) may be replaced by the less restrictive condition

(ii') a modular right ideal $\mathfrak{M}$ of $\mathfrak{A}$ such that $M \neq A$ and $M^* \neq A^*$ is either maximal or is contained in a normal right ideal $\mathfrak{N}$ such that $N \neq A$ and $\mathfrak{N} \neq \mathfrak{M}$.

In the proof of our theorem, we claim that the modular right ideal $\mathfrak{M}$ is maximal in $\mathfrak{A}$. For this to happen, it is necessary that $M \neq A$ (Patterson [1], § 10); condition (i) is not sufficient to ensure this. For example, consider the pseudo-ring (Z, Q) , where Z is the ring of integers and Q the Z -module of rational numbers; (Z, Q) has no maximal right ideals and so coincides with its own radical. Now $1 \in Z$ is not right quasi-regular, while condition (i) is vacuously satisfied.

We next show that condition (ii) in [1], Theorem 6, may be replaced by (ii') as above. Following the proof given in [1], we arrive at a normal right ideal $\mathfrak{N}$ satisfying $\mathfrak{M} \subset \mathfrak{N} \subset \mathfrak{A}$. If $b^* \notin N$, then $\mathfrak{N} \in \mathfrak{S}$ which contradicts the maximality of $\mathfrak{M}$ in $\mathfrak{S}$; thus $b^* \in N$ whence we obtain a contradiction as in [1].

We note that condition (ii') is less restrictive than (ii). For instance, it may be verified that the pseudo-ring (Z, Z) satisfies (ii'); however, consideration of the right ideal $(0, pZ)$, where p is prime, shows that (Z, Z) does not satisfy (ii).

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